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Limiting Gravity Waves in Water of Finite Depth

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LIMITING GRAVITY WAVES IN WATER OF FINITE DEPTH

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CONTENTS

Progressive, irrotational gravity waves of constant form exist as a two-parameter family. The first parameter, the ratio of mean depth to wavelength, varies from zero (the solitary wave) to infinity (the deep-water wave). The second parameter, the wave height or amplitude, varies from zero (the infinitesimal wave) to a limiting value dependent on the first parameter. For limiting waves the wave crest ceases to be rounded and becomes angled, with an included angle of 120°.

Most methods of calculating finite-amplitude waves use either a form of series expansion or the solution of an integral equation. For waves nearing the limiting amplitude many terms (or nodal points) are needed to describe the wave form accurately. Consequently the accuracy even of recent solutions on modern computers can be improved upon, except at the deep-water end of the range.

The present work extends an integral equation technique used previously in which the angled crest of the limiting wave is included as a specific term, derived from the well known Stokes corner flow. This term is now supplemented by a second term, proposed by Grant in a study of the flow near the crest. Solutions comprising 80 terms at the shallow-water end of the range, reducing to 20 at the deep-water end, have defined many field and integral properties of the flow to within 1 to 2 parts in 10^6. It is shown that without the new crest term this level of accuracy would have demanded some hundreds of terms while without either crest term many thousands of terms would have been needed.

The practical limits of the computing range are shown to correspond, to working accuracy, with the theoretical extremes of the solitary wave and the deep-water wave. In each case the results agree well with several previous accurate solutions and it is considered that the accuracy has been improved. For example, the height: depth ratio of the solitary wave is now estimated to be 0.833 197 and the height: wavelength ratio of the deep-water wave to be 0.141 063.

The results are presented in detail to facilitate further theoretical study and early practical application. The coefficients defining the wave motion are given for 22 cases, five of which, including the two extremes, are fully documented with tables of displacement, velocity, acceleration, pressure and time.

Examples of particle orbits and drift profiles are presented graphically and are shown for the extreme waves to agree very closely with simplified calculations by Longuet-Higgins.

Finally, the opportunity has been taken to calculate to greater accuracy the long-term Lagrangian-mean angular momentum of the maximum deep-water wave, according to the recent method proposed by Longuet-Higgins, with the conclusion that the level of action is slightly above the crest.

1. Introduction

The theory of infinitesimal periodic waves was set down in the 19th century as also was the first extension by Stokes (1847) to waves of finite amplitude. For so-called shallow-water waves, whose depth is only a small proportion of the wavelength, finite amplitudes are better treated by the cnoidal theory which is elegantly presented by Benjamin & Lighthill (1954) and has recently been extended to fifth and ninth orders by Fenton (1979). These theories become inadequate, however, when the amplitude is a significant proportion of its maximum possible value. At the maximum value itself the crest of the steady-flow description of the wave, obtained by superimposing the celerity, is at the total energy level. The surface streamline necessarily has a stagnation point there and, as shown by Stokes (1880), the crest is angled rather than rounded, with an included angle of 120°.

It is to be expected that as the limiting crest form is approached the number of Stokes-type terms needed to describe it adequately will become very large. Before the advent of digital computers the task of extending the theory beyond a very modest order was prohibitive, practical limits being reached by, for example, the fifth-order solution of De (1955). With the aid of a computer Schwartz (1974) later performed the series development mechanically and found that Stokes's original formulation was fundamentally unable to extend the wave height beyond a definite limit, well short of the maximum. He did, however, recast the method by interchanging dependent and independent variables and using a more suitable expansion parameter. In this way he obtained some solutions of high accuracy, one extending to 117th order.

Several valuable solutions for maximum waves have been obtained by imposing a crest of the correct form and building other standard terms around it to account for the remainder of the profile. In particular Yamada (1957*a*, *b*) has solved the deep-water and solitary wave in this way while Lenau (1966) has solved the solitary wave. The present work was started with the aim of improving the accuracy of this type of method and extending it to all depth: wavelength ratios and to waves of less than maximum height. It was found that to improve the accuracy significantly it was necessary to use two terms rather than one to describe the crest singularity. The second term was suggested by the work of Grant (1973) who showed that the single term proposed by Stokes could not adequately account for the flow a short distance from the crest. With these two terms included in an integral equation formulation a set of solutions has been obtained for all depth: wavelength ratios whose accuracy appears to be superior to any previous maximum wave solutions. In relation to the accuracy achieved the number of terms used is modest, ranging from 80 at the shallow-water end of the computing range to 20 at the deep-water end.

Section 2 sets out the formulation of the problem and § 3 discusses the form of the crest of the limiting wave.

As is usual with numerical methods, the solutions generated are approximate only and have a small residual error in the dynamic free-surface boundary condition. They may, however, be regarded as exact solutions of a periodic irrotational flow with a small but non-zero surface pressure. Section 4 shows how integrals of this pressure may be included as explicit error terms in revised forms of the several identities relating integral properties of waves presented by Longuet-Higgins (1974, 1975). These error terms are later shown to be very small in all of the present solutions.

Sections 5 and 6 describe the algorithm, § 7 gives the basic results in the form of the coefficients defining each solution, and § 8 shows that the overall accuracy is generally better than that of previous solutions. Sections 9 and 10 show that the limits of our range are to computing accuracy identical with the solitary wave and deep-water wave respectively. The height, speed and integral properties of each are compared with several previous accurate solutions. The agreement is generally good and the accuracy is considered to have been improved. The opportunity has also been taken to repeat to greater accuracy a recent calculation by Longuet-Higgins (1980) of the level of action of the maximum deep-water wave, which is shown to be slightly above the crest.

The objective of the work has been not only to define limiting waves accurately but also to present the results in a form permitting their ready application. Accordingly, § 11 gives detailed tabulations for the solitary wave, deep-water wave and three intermediate waves, and finally § 12 illustrates the use of the tables to determine particle paths and drift profiles. It is shown that recent estimates of these made by Longuet-Higgins (1979), from very simple approximations to the solitary and deep-water waves, are remarkably accurate.

2. Formulation of the problem

We are to consider progressive, symmetrical, irrotational, inviscid waves propagated without change of form in liquid of uniform and finite depth. Following normal practice, we superimpose upon the flow a velocity equal and opposite to the wave celerity, when the motion becomes steady.

Figure 1 shows this steady flow in the physical plane of $z = x+iy$. Next, to remove the initially unknown free-surface boundary, we interchange dependent and independent variables and define the flow in the plane of the complex potential $\chi = \phi+i\psi$ as shown in figure 2. The potential χ is normalized such that ψ ranges from zero at the surface to -2 at the bed, the range of ϕ then being from zero at the crest to $-\lambda$ at the following crest. Solutions are to be sought for the full range of λ from zero (deep-water wave) to infinity (solitary wave).

To normalize the physical variables the flow is referred to the simple standard case of the lowest mode of infinitesimal wave motion that would occupy the complex potential domain of figure 2. With reference to figure 1, the origin is taken above the crest of this wave at the level of its total energy line, x being measured in the direction of the steady motion and y downwards. The mean surface level of the infinitesimal motion is then defined to be at $y = F^2$, where F^2 is the square of a Froude number, and the bed is given by $y = 2+F^2$, so that the mean velocity of the steady motion is unity and its wavelength is λ. We introduce the variables u and v to denote the components of velocity in the x and y directions respectively. The ratio $4\pi/\lambda$ is now defined as d. Our variable d coincides with d as defined by Cokelet (1977) and $-\ln r_0$ as defined by Schwartz (1974).

The solution of the infinitesimal motion of semi-amplitude a is given for example by Lamb (1932) and is in the present notation

$$z = x+iy = -\chi+iF^2-ia\sinh d(1-\tfrac{1}{2}i\chi)/\sinh d. \tag{2.1}$$

The variables x and y satisfy Laplace's equation within the flow domain while y satisfies the required conditions at the fixed boundaries, namely

$$y = 2+F^2 \quad (\psi = -2), \qquad \partial y/\partial\phi = 0 \quad (\phi = 0, -\lambda). \tag{2.2}$$

If p denotes the ratio of pressure to density, Bernoulli's equation takes the form

$$p+\tfrac{1}{2}(u^2+v^2) - \tfrac{1}{2}y/F^2 = 0. \tag{2.3}$$

The free-surface condition to be satisfied by the solution is obtained by setting $p = 0$ in (2.3), which may then be written as

$$\operatorname{Im}(z)\,|dz/d\chi|^2 = F^2, \quad \psi = 0. \tag{2.4}$$

Equations (2.1) and (2.4) are compatible if

$$F^2 = (1/d)\tanh d, \tag{2.5}$$

provided also that powers of a beyond the first are neglected.

Since (2.1) constitutes a small perturbation of a uniform stream it is convenient to isolate the perturbation, by means of the new variable ζ, from the uniform stream, z_0. Then

$$\left.\begin{aligned} z &= z_0+\zeta, \\ \text{where} \quad z_0 &= x_0+iy_0 = -\chi+iF^2 \end{aligned}\right\} \tag{2.6}$$

and
$$\zeta = \xi+i\eta = -ia\sinh d(1-\tfrac{1}{2}i\chi)/\sinh d. \tag{2.7}$$

For a wave motion of finite height H lying within the complex potential domain of figure 2 the wavelength will be L rather than λ and the mean depth will be h rather than 2. We define the ratio λ/L as c. The total energy line will also differ from that for the infinitesimal wave. We retain F^2, defined by (2.5), as a normalizing parameter, so that the acceleration due to gravity is $1/2F^2$ in our scaling. If α is defined such that the total energy line is a distance $2+\alpha F^2$ above the bed the free-surface boundary condition becomes

$$[\operatorname{Im}(z)+(\alpha-1)F^2]\,|dz/d\chi|^2 = F^2, \quad \psi = 0. \tag{2.8}$$

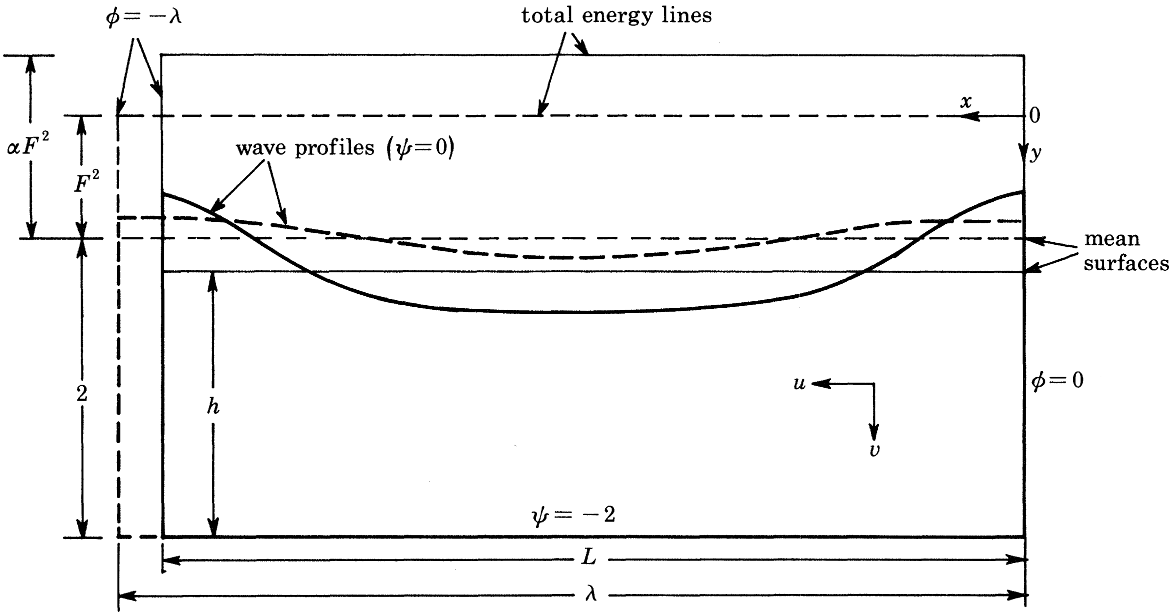

FIGURE 1. The z-plane. — — — —, — — — infinitesimal wave; ————, ——— finite wave.

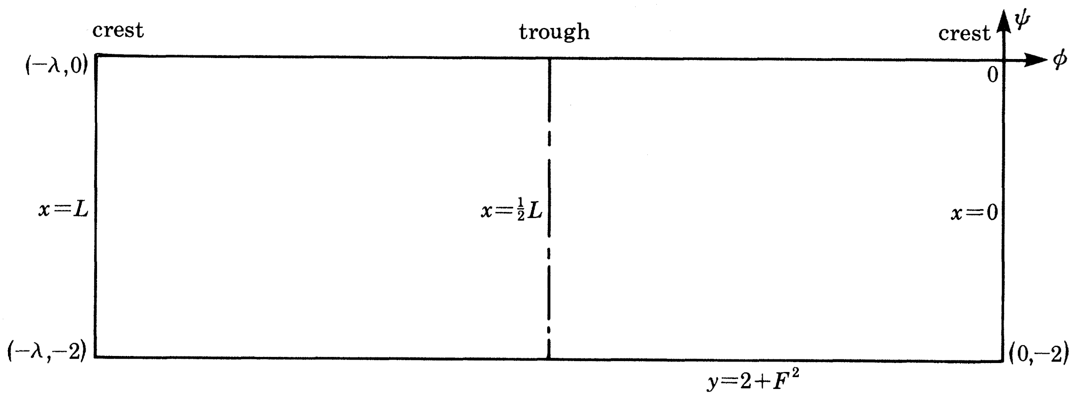

FIGURE 2. The χ-plane.

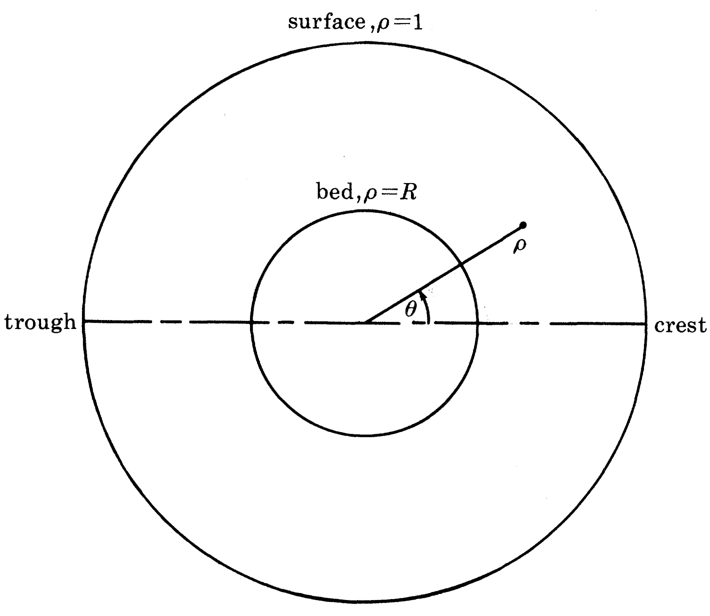

FIGURE 3. The τ-plane.

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Figure 1 shows the outline and datum lines of the finite wave compared with those of the infinitesimal wave.

To take advantage of the periodicity of the motion we next introduce the transformation

$$\chi = \mathrm{i}(2/d)\ln\tau, \quad \tau = \rho\,\mathrm{e}^{\mathrm{i}\theta}. \tag{2.9}$$

With reference to figure 3, the flow field is bounded in the τ-plane by the concentric circles $\rho = \mathrm{e}^{-d} = R$ (bed) and $\rho = 1$ (surface), while the wave crest occurs at $\tau = 1$ and the trough at $\tau = -1$. Limiting cases are given by $R = 0$ (deep-water wave) and $R = 1$ (solitary wave).

The uniform stream z_0 is related to τ by

$$z_0 = -\mathrm{i}(2/d)\ln\tau + \mathrm{i}F^2, \tag{2.10}$$

and the imaginary part η of the perturbation variable ζ is required to be symmetrical about $\theta = 0$, to satisfy Laplace's equation

$$\nabla^2\eta = 0, \quad R \leqslant \rho \leqslant 1, \tag{2.11}$$

the bed condition

$$\eta = 0, \quad \rho = R, \tag{2.12}$$

and the surface condition

$$(\alpha F^2 + \eta)\left[\left(1 - \tfrac{1}{2}d\frac{\partial\eta}{\partial\rho}\right)^2 + \left(\tfrac{1}{2}d\frac{\partial\eta}{\partial\theta}\right)^2\right] = F^2, \quad \rho = 1. \tag{2.13}$$

All conditions except the surface condition (2.13) are satisfied by solutions of the form

$$\zeta_0 = -\mathrm{i}(1/d)\ln(\tau/R), \tag{2.14}$$

which gives a uniform flow in the χ-plane, and

$$\zeta_m = -\frac{\mathrm{i}}{1-R^{2m}}\left[\tau^m - \left(\frac{R^2}{\tau}\right)^m\right], \quad m = 1, 2, \ldots, \infty, \tag{2.15}$$

which are modes of the periodic motion whose fundamental is given by (2.7). If therefore we choose a linear combination of N component solutions given by

$$\zeta = a_0\zeta_0 + a_1\zeta_1 + \ldots + a_{N-1}\zeta_{N-1} \tag{A}$$

we may in principle determine the unknown coefficients $a_0, a_1, \ldots, a_{N-1}$ by satisfying the surface condition (2.13), with α assumed known, at N discrete nodal points on the circumference of the outer circle $\rho = 1$. In view of the symmetry these points should be taken only over the upper semicircle, $0 \leqslant \theta \leqslant \pi$.

When a solution has been obtained in this way condition (2.13) will in general be violated to some extent between the nodal points. We may, however, view our solution as an exact solution of a flow with a non-zero surface pressure $p_s(\theta)$ given by

$$p_s(\theta) = \frac{1}{2}\left[\frac{\eta}{F^2} + \alpha - \frac{1}{(1-\frac{1}{2}d\,\partial\eta/\partial\rho)^2 + (\frac{1}{2}d\,\partial\eta/\partial\theta)^2}\right] \tag{2.16}$$

evaluated at $\rho = 1$. For comparison with other computed solutions it is also convenient to convert p_s to an equivalent surface elevation error and express it as a proportion of the wave height H. We therefore define the error variable $\epsilon(\theta)$ given by

$$\epsilon(\theta) = 2F^2 p_s(\theta)/H. \tag{2.17}$$

We shall be concerned in the discussion with the maximum modulus over a wavelength of $p_s(\theta)$, to be denoted by $\hat{p}_s$, and the root mean square of $\epsilon(\theta)$, to be denoted by ϵ^*.

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3. The crest of the limiting wave

When the wave reaches its maximum height the crest reaches the total energy line of the steady motion and also becomes angled rather than rounded, with an included angle of 120°. The flow in the vicinity of the crest, the Stokes (1880) corner flow, has been discussed also for example by Milne-Thomson (1968) and Longuet-Higgins (1979). With Z defined to move the space origin temporarily to the crest, the flow is given by

$$\chi = -\mathrm{i}(\sqrt{2}/3F)\,[-\mathrm{i}z+(\alpha-1)\,F^2]^{\frac{3}{2}} = -\mathrm{i}(\sqrt{2}/3F)\,(-\mathrm{i}Z)^{\frac{3}{2}}. \tag{3.1}$$

Thus, if we write

$$Z = \mathrm{i}(3\sqrt{2}\,F/d)^{\frac{2}{3}}\,(1-\tau)^{\frac{2}{3}}, \tag{3.2}$$

we obtain the correct form at the crest without singularities elsewhere in the τ-plane. Next, from Milne-Thomson's circle theorem, we deduce that a flow complying with (3.2) at the crest and also fulfilling the correct boundary condition at the bed, $\rho = R$, is

$$\zeta = \mathrm{i}(3\sqrt{2}\,F/d)^{\frac{2}{3}}\,[(1-\tau)^{\frac{2}{3}}-(1-R^2/\tau)^{\frac{2}{3}}]. \tag{3.3}$$

Grant (1973) showed that the crest flow could not be described simply by a term of the form (3.1) and sought to develop an expansion for the flow near the crest in the form (in our notation, but with Grant's leading coefficient being preserved within the bracket)

$$Z = -\mathrm{i}(\sqrt{2}\,F)^{\frac{2}{3}}\,[-(\tfrac{3}{2})^{\frac{2}{3}}\,(\mathrm{i}\chi)^{\frac{2}{3}}+b(\mathrm{i}\chi)^{\mu}\ldots]. \tag{3.4}$$

He found that b could not be determined from local considerations alone while μ was a root of the equation

$$-\tan\tfrac{1}{2}\pi\mu = (4+3\mu)/(3\sqrt{3}\,\mu), \tag{3.5}$$

which, if $-\frac{1}{2}\pi(\mu+\frac{1}{3})$ is written as K, takes the simpler form (Longuet-Higgins & Fox 1977)

$$K\tan K = -\tfrac{1}{2\sqrt{3}}\pi. \tag{3.6}$$

The first root of (3.5) greater than $\frac{2}{3}$ is $\mu = 1.469$. Eight further terms in the expansion (3.4) based on this root have been calculated in terms of b and are given in Appendix 1.

To accommodate the various forms of components of ζ that can be expected to feature in solutions of maximum and less-than-maximum waves we define the general variable $\zeta_{m,A,\nu}$ given by

$$\zeta_{m,A,\nu} = \frac{\mathrm{i}\{[A^{-1}-\tau^m]^{\nu}-[A^{-1}-(R^2/\tau)^m]^{\nu}\}}{[A^{-1}-R^{2m}]^{\nu}-[A^{-1}-1]^{\nu}}. \tag{3.7}$$

The chosen denominator normalizes the function by giving $\eta = -1$ at $\tau = 1$, as for the basic modes of (2.15). A ranges from 0 to 1 and allows the singularity to be taken away from the surface for waves of less than maximum amplitude. As A tends to zero the function $\zeta_{m,0,\nu}$ degenerates to the form of (2.15), which for brevity we will continue to denote by ζ_m.

4. Integral properties of waves

Longuet-Higgins (1975) and others have defined several integral properties of a progressive wave motion and derived certain identities relating them. The solutions to be presented in this paper are approximations to the required wave motion but, as has been shown, may be regarded as exact solutions of a flow with a non-zero surface pressure $p_s(\theta)$. It is therefore valuable to rework Longuet-Higgins's analysis with due allowance for p_s and derive a revised set of identities

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involving integrals of p_s. The magnitude of these integrals will be a useful indication of the closeness of our solutions to the true wave solution with identically zero p_s.

We use a subscript s to denote a value at the surface. Throughout this section the limits of integration are understood to be 0, L for x and y_s, $2+F^2$ for y, unless stated otherwise.

The mean value of η_s over the wavelength is $\bar{\eta}_s$ given by

$$\bar{\eta}_s = (1/L)\int \eta_s \,\mathrm{d}x = \bar{y}_s - F^2 \tag{4.1}$$

and Longuet-Higgins's excess mass M is then defined in our notation by

$$M = \int (\bar{\eta}_s - \eta_s)\,\mathrm{d}x = \int (\bar{y}_s - y_s)\,\mathrm{d}x = 0. \tag{4.2}$$

If we superimpose on the steady motion considered so far a velocity $-c$ we obtain a wave moving in the direction of negative x with velocity components $\tilde{u}$, $\tilde{v}$, where

$$\tilde{u} = \partial\phi/\partial x + c, \quad \tilde{v} = -\partial\phi/\partial y, \tag{4.3}$$

$\tilde{u}$ being taken as positive in the direction of motion of the wave. Longuet-Higgins defines, for an integral taken along the bed,

$$C = \int \tilde{u}\,\mathrm{d}x = \int (\partial\phi/\partial x + c)\,\mathrm{d}x, \tag{4.4}$$

which is zero on account of the choice $-c$ for the superimposed velocity. He continues by defining the parameters I, T, V representing the mean densities over the wavelength of momentum, kinetic energy and potential energy respectively. Thus

$$I = (1/L)\iint \tilde{u}\,\mathrm{d}y\,\mathrm{d}x, \tag{4.5}$$

$$T = (1/L)\iint \tfrac{1}{2}(\tilde{u}^2+\tilde{v}^2)\,\mathrm{d}y\,\mathrm{d}x, \tag{4.6}$$

$$V = (1/2F^2)\left[(-1/L)\iint y\,\mathrm{d}y\,\mathrm{d}x + h(2+F^2-\tfrac{1}{2}h)\right]. \tag{4.7}$$

Two further parameters are S_{xx}, the radiation stress, and E (denoted F by Longuet-Higgins), the mean energy flux, which are defined in the present notation by

$$S_{xx} = (1/L)\iint (p+\tilde{u}^2)\,\mathrm{d}y\,\mathrm{d}x - (h/2F)^2, \tag{4.8}$$

$$E = (1/L)\iint [p+\tfrac{1}{2}(\tilde{u}^2+\tilde{v}^2)-(y/2F^2)]\,\tilde{u}\,\mathrm{d}y\,\mathrm{d}x + (2+F^2-h)\,I/2F^2. \tag{4.9}$$

Longuet-Higgins's first identity is

$$2T = cI, \tag{4.10}$$

a kinematic relation which continues to hold for the present solutions. The remaining identities involve momentum and energy balances which will include extra terms arising from the non-zero surface pressure p_s in our solutions. We define the integral parameters

$$P_1 = (1/L)\int p_s\,\mathrm{d}x, \tag{4.11}$$

$$P_2 = (1/L)\int p_s(\bar{\eta}_s - \eta_s)\,\mathrm{d}x = (1/L)\int p_s(\bar{y}_s - y_s)\,\mathrm{d}x \tag{4.12}$$

and

$$P_3 = \frac{1}{L}\int\left(\int_x^{\frac{1}{2}L} p_s \frac{\mathrm{d}y_s}{\mathrm{d}x}\,\mathrm{d}x\right)\mathrm{d}x. \tag{4.13}$$

We also introduce σ_b, σ_t to denote respectively the standard deviations (in space) of the velocity at the bed and the velocity beneath the trough. Thus

$$\sigma_b^2 = (1/L)\int \tilde{u}^2\,dx \tag{4.14}$$

evaluated along the bed and, with h_t denoting the trough depth,

$$\sigma_t^2 = \frac{1}{h_t}\int\left(\frac{2}{h_t}+\frac{\partial\phi}{\partial x}\right)^2 dy \tag{4.15}$$

evaluated beneath the trough.

The definition of S_{xx} may be rearranged and the momentum flux at each vertical section related to the momentum flux at the trough in the steady flow. This demonstrates the effect of p_s and leads to

$$S_{xx} = 2cI-(h^2-h_t^2)/4F^2-c^2h+\tfrac{1}{2}h_t[(2/h_t)^2+(c-\tilde{u}_{st})^2+\sigma_t^2]+P_3, \tag{4.16}$$

where $\tilde{u}_{st}$ is the value of $\tilde{u}_s$ at the trough. Repetition of Longuet-Higgins's analysis with allowance for the surface pressures then gives the following revised identities:

$$3V = 4T-S_{xx}+2Bh-hP_1-P_2 \tag{4.17}$$

and

$$E = (3T-2V)c+(I+ch)B-(hP_1+P_2)c, \tag{4.18}$$

where

$$B = (2-h)/2F^2+\tfrac{1}{2}(\alpha-c^2) = \tfrac{1}{2}\sigma_b^2+P_1. \tag{4.19}$$

The terms involving P_1, P_2, P_3 in (4.16)–(4.19) are the error terms associated with our solutions.

The solitary wave

For the theoretical extreme of the solitary wave with infinite wavelength Longuet-Higgins (1974) proposed modified definitions which we shall denote in this paper by a subscript ∞. The present algorithm cannot explicitly compute the solitary wave since the computing domain degenerates to the circumference of the unit circle. To compare our results for long periodic waves, at the limit of the present method, with the true solitary wave it is useful to relate the two sets of parameters. We therefore give the definitions of the solitary wave parameters and accompany them with relations which, although not exact, can be satisfied to an arbitrarily fine computing tolerance. The solitary wave has a celerity c_∞, a velocity field $\tilde{u}_\infty$, $\tilde{v}_\infty$ and an undisturbed depth h_∞ at infinity, where the velocity is zero. The corresponding periodic wave has a non-zero average velocity $\tilde{U}$ beneath the trough, so that

$$\tilde{U} = c-2/h_t \approx c-c_\infty. \tag{4.20}$$

It is assumed that, to computing accuracy, the velocity beneath the trough is sensibly uniform, the pressure distribution is hydrostatic and σ_t^2 has vanished. The following relations then apply:

$$M_\infty = \int_{-\infty}^{\infty}(2+F^2-h_\infty-y_s)\,dx \approx \int(2+F^2-h_t-y_s)\,dx = L(h-h_t); \tag{4.21}$$

$$C_\infty = \int_{-\infty}^{\infty}\tilde{u}_\infty\,dx \approx \int(\tilde{u}-\tilde{U})\,dx = -L\tilde{U}, \tag{4.22}$$

with the integral taken along the bed as before;

$$I_\infty = \int_{-\infty}^{\infty}\int \tilde{u}_\infty \,\mathrm{d}y\,\mathrm{d}x \approx \iint (\tilde{u}-\tilde{U})\,\mathrm{d}y\,\mathrm{d}x = L(I-\tilde{U}h); \tag{4.23}$$

$$T_\infty = \int_{-\infty}^{\infty}\int \tfrac{1}{2}(\tilde{u}_\infty^2+\tilde{v}_\infty^2)\,\mathrm{d}y\,\mathrm{d}x \approx \iint \tfrac{1}{2}[(\tilde{u}-\tilde{U})^2+\tilde{v}^2]\,\mathrm{d}y\,\mathrm{d}x$$
$$= L(T-\tilde{U}I+\tfrac{1}{2}\tilde{U}^2h); \tag{4.24}$$

$$V_\infty = (1/4F^2)\int_{-\infty}^{\infty}(2+F^2-h_\infty-y_s)^2\,\mathrm{d}x$$
$$\approx (1/4F^2)\int (2+F^2-h_t-y_s)^2\,\mathrm{d}x = L[V+(h-h_t)^2/4F^2]. \tag{4.25}$$

With these approximations and the earlier results of this section Longuet-Higgins's identities for solitary waves may be reconstructed. We conclude that we expect our computed results to satisfy

$$I_\infty - c_\infty M_\infty \approx 0, \tag{4.26}$$

$$2T_\infty - c_\infty(I_\infty - h_\infty C_\infty) \approx 0, \tag{4.27}$$

$$3V_\infty - (c_\infty^2 - h_\infty/2F^2)\,M_\infty \approx -L(hP_1+P_2+P_3). \tag{4.28}$$

To test the extent to which a computed long periodic wave represents a true solitary wave we therefore calculate the left-hand sides of (4.26)–(4.28). They will differ from zero according to the general computing accuracy, the magnitude of P_1, P_2, P_3, and the extent to which the assumptions leading to (4.21) to (4.25) are violated.

We note finally that Longuet-Higgins normalizes the solitary wave parameters by taking h_∞ and gravity $(1/2F^2)$ as unity. When comparing our values with previous results in § 9 and table 5 we shall scale them accordingly.

5. The computing algorithm

Initial work was done by the author in 1970, using a trial solution in the form (A) of § 2. The N nodal points were distributed evenly on the upper semicircle $\rho = 1$ in the τ-plane at the points $\theta = (2k-1)\,\pi/2N$, $k = 1, 2, \ldots, N$. A constraint was applied to the amplitude of the wave being computed by assigning a fixed value to the leading coefficient a_1. The N unknowns to be found were then α, a_0, a_2, a_3, …, a_{N-1}.

For a trial set of coefficients the free-surface error $\epsilon(\theta)$ (equation (2.17)) was evaluated at each nodal point to give an error vector $\boldsymbol{\varepsilon} = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_N\}$. Each trial coefficient was then perturbed by a small quantity and the errors were recalculated and used to form a rate-of-change matrix Δ with elements

$$\frac{\partial\epsilon_k}{\partial\alpha},\quad \frac{\partial\epsilon_k}{\partial a_0},\quad \frac{\partial\epsilon_k}{\partial a_2},\quad \ldots,\quad \frac{\partial\epsilon_k}{\partial a_{N-1}},\quad k = 1,\ 2,\ \ldots,\ N.$$

Estimates of corrections to the coefficients were evaluated from $-\Delta^{-1}\boldsymbol{\varepsilon}$ and the cycle was repeated until convergence had been achieved to an acceptable tolerance.

This algorithm converged well and yielded useful solutions for waves up to about half of maximum amplitude over a wide range of R, with N taking values up to 21, the maximum feasible on the small computer (ICL 1901A) available at the time. For recent work, designed to cover all amplitudes, the program has been made more flexible and has been run on the ICL 1904S at the Hydraulics Research Station and the CDC 7600 at the University of London Computer Centre. In addition to a subroutine computing the general set of component functions (3.7) and their derivatives, the program has the facility for varying both the distribution of the nodal points and the choice of fixed and 'floating' coefficients.

6. Computation of maximum waves

The first attempts to compute maximum waves were made with the addition of a term $s\zeta_{1,1,\frac{2}{3}}$ to scheme (A). The form used was

$$\zeta = s\zeta_{1,1,\frac{2}{3}} + a_0\zeta_0 + a_1\zeta_1 + \ldots + a_{N-2}\zeta_{N-2}, \tag{B}$$

with the nodal points taken at $\theta = k\pi/N$, $k = 0, 1, 2, \ldots, N$. With the first nodal point then at the crest and the singularity set at the surface ($A = 1$) the constraint on amplitude previously applied by fixing a_1 became redundant. Thus the $N+1$ nodal points served to find the $N+1$ coefficients $\alpha, s, a_0, a_1, \ldots, a_{N-2}$. In an alternative scheme s was fixed at the value corresponding to the Stokes corner flow given by (3.3) and a further coefficient a_{N-1} was added. Although these schemes gave promising results, with N still no larger than 21, it was clear that something more would be needed if accuracy were to be significantly advanced without a disproportionate increase in computing effort.

Attention was therefore paid to the further terms proposed by Grant, and a component $q\zeta_{1,1,\mu}$ was added to the trial solution. The corresponding extra nodal point was, after some experimentation, placed at a very small distance θ_c from the crest with the remaining nodes continuing to be uniformly distributed between crest and trough. This scheme gave a marked increase in accuracy and is the basis on which the results to be presented have been computed.

The coefficient s of the main singularity was allowed to float in the iteration since this gave better overall accuracy than if it were fixed according to (3.3). The penalty for allowing s to float in this way is a computed solution whose acceleration at the crest differs from the theoretical value for the Stokes corner flow of $\frac{1}{2}g$, or $1/4F^2$ in our notation (Longuet-Higgins & Fox 1977). However, the discrepancy is very localized and is shown in Appendix 2 not to have significant consequences.

Before the above scheme was finally accepted many tests were made in an effort to improve the accuracy further, but without significant success. Early experiments on nodal point spacing suggested that uniform spacing could not easily be improved upon. The auxiliary crest node was initially placed midway between the first two existing nodes and then moved progressively nearer the crest. For $N = 21$ the position $\theta_c = \frac{1}{420}\pi$ seemed to be near-best but the accuracy was found not to be critical over a modest range around this value. The position $\theta_c = \frac{1}{420}\pi$ was therefore used for many early runs although it was subsequently changed to $\frac{1}{280}\pi$, as explained in §7. The scheme appears reasonable in that the two terms $s\zeta_{1,1,\frac{2}{3}}$ and $q\zeta_{1,1,\mu}$ are being used to define the crest flow, and the two nodes near the crest can be expected to have the dominant influence on these terms. The nodes are not, however, so close as to lose their independence of influence on the error vector, which would tend to make the matrix Δ singular and impair the iteration.

Terms beyond $q\zeta_{1,1,\mu}$ in Grant's series were tried but gave no obvious further improvement. If the coefficients were treated as unknowns and iterated, the convergence became ineffective because (see Appendix 1) the relevant exponents were 2.27, 3.07, etc., which were insufficiently independent of the basic terms $a_m \zeta_m$. If on the other hand the coefficients were tied to the unknown q, in accordance with the ratios shown in table A 1, a negligible improvement resulted, even with eight further terms included. This result, although disappointing, is again reasonable because in seeking a result which is accurate in an overall average sense we must expect, as has already occurred with s, some compromising of exact relations applicable to a special point in the flow. This reasoning raised in turn the question of whether $\mu = 1.469$, Grant's theoretical index, was necessarily the best for our overall solution. Small variations were accordingly tried without dramatic effect and the conclusion was reached that no other value was obviously better.

The point was thus reached, after many experiments attempting to draw a balance between theoretical guidance and empirical trial-and-error, when the following scheme was accepted for computing maximum waves:

$$\left.\begin{array}{ll} & \zeta = s\zeta_{1,1,\frac{2}{3}} + q\zeta_{1,1,\mu} + a_0\zeta_0 + a_1\zeta_1 + \ldots + a_{N-2}\zeta_{N-2} \\ \text{with} & N+2 \text{ unknowns}, \quad \alpha, s, q, a_0, a_1, \ldots, a_{N-2} \\ \text{and} & N+2 \text{ nodal points}, \quad \theta = \theta_c \quad \text{and} \quad \theta = k\pi/N, \quad k = 0, 1, 2, \ldots, N. \end{array}\right\} \quad \text{(C)}$$

The appropriate choice of N and the final choice of θ_c will be discussed in §7.

The two leading terms in this scheme may, from their definition in (3.7), be expanded as identical series in powers of τ and R^2/τ. Hence, in view of (2.15), we may write as an equivalent scheme

$$\zeta = \sum_{m=1}^{\infty} s_m \zeta_m + \sum_{m=1}^{\infty} q_m \zeta_m + \sum_{m=0}^{N-2} a_m \zeta_m = \sum_{m=0}^{\infty} A_m \zeta_m, \quad (6.1)$$

which corresponds to scheme (A) of §2 except for the infinite limit. When a solution has been computed according to (C) the total coefficient A_m of ζ_m may be constructed from the relations

$$\left.\begin{array}{l} A_0 = a_0, \\ (-)^m A'_m = -s\left[\dfrac{1-R^{2m}}{(1-R^2)^{\frac{2}{3}}}\right]\dbinom{\frac{2}{3}}{m} - q\left[\dfrac{1-R^{2m}}{(1-R^2)^{\mu}}\right]\dbinom{\mu}{m}, \quad m = 1, 2, \ldots, \infty, \\ A_m = A'_m + a_m, \quad m = 1, 2, \ldots, N-2, \\ A_m = A'_m, \quad m = N-1, N, \ldots, \infty. \end{array}\right\} \quad (6.2)$$

The coefficients A_m will be considered in the discussion of the accuracy of the solutions.

We note that when (2.13) is applied at the crest node the result is

$$\alpha F^2 = s + q + \sum_{m=0}^{N-2} a_m = \sum_{m=0}^{\infty} A_m, \quad (6.3)$$

a simple relation between α and either set of coefficients.

7. Results – computed coefficients

The algorithm described has successfully computed solutions for values of d ranging from 0.1 to 10.0, giving maximum waves whose depth: wavelength ratio varies from 0.016 to 1.60. It will be shown that, to present computing accuracy, the upper limit of this range gives the solution for the deep-water wave while for $d \leqslant 0.2$ the solution corresponds to that of the solitary wave.

For each value of d the choice of N was found to have an important influence not only on accuracy but also on the ability of the algorithm to converge. The behaviour was also different on either side of the point $d \approx 0.5$. For $d \leqslant 0.5$, the shallow-water end of the range, more terms were needed for a given accuracy and it was found, surprisingly, that convergence was lost for any even value of N exceeding about 40. If N remained odd, however, its value could be freely increased with strong convergence and progressively greater accuracy. The limit for a reasonable run on the CDC 7600 was given by $N = 79$ which was used for the lowest values of d. The solutions gave a positive set of coefficients $a_2, a_3, \ldots, a_{N-2}$ decreasing monotonically to zero.

When $d > 0.5$ accuracy no longer increased indefinitely with N but reached a peak at an optimum value ranging from 41 at $d = 0.5$ to 19 at $d = 10.0$. Within this range both odd and even values of N gave satisfactory solutions although even values again caused difficulty at the largest values of d. The reasons for these features of the method, which are discussed further in Williams (1981), have not yet been investigated in detail since, once identified, they did not impede the production of the solutions required.

Since at the deep-water end of the range, where N is small, the solutions could not be adjusted finely enough by varying N, attention was paid instead to θ_c, the position of the second crest node. This led to the revised choice of $\frac{1}{280}\pi$ as the best value, which was accordingly adopted throughout, in preference to the original value of $\frac{1}{420}\pi$, for the results to be presented.

Table 1 gives the computed coefficients for 22 values within the range $0.2 \leqslant d \leqslant 10.0$. They are tabulated to eight decimal places for $d \leqslant 1.6$ and to nine places for the remaining values. This ensures that residual errors are small compared with the intrinsic errors of the method which are responsible for the non-zero surface pressure p_s and its integrals P_1, P_2, P_3. Although not included in table 1, α may readily be computed from (6.3).

To achieve the required accuracy generally between five and thirty iterations were needed, depending on the starting values available and the rate of convergence. The time per iteration on the CDC 7600 varied approximately as $N^{2.84}$ and was about 5 seconds for $N = 35$.

8. Accuracy compared with previous solutions

Since each computed result is an exact solution of a periodic flow with a small non-zero surface pressure p_s, the magnitude of p_s and its integrals provides the readiest explicit indication of accuracy. Table 2 shows, for each solution given in table 1, the values of $\hat{p}_s$ and ϵ^* as defined in § 2. For each solution ϵ^* is less than 2×10^{-6}.

The pressure integrals P_1, P_2, P_3 defined in (4.11)–(4.13) are less than 5×10^{-8} with the single exception of the solution for $d = 2.5$, where $P_1 = 10^{-7}$. It is thus immediately clear from the error terms in the revised integral identities (4.16)–(4.19) that the solutions can be expected to satisfy Longuet-Higgins's original identities to about six-decimal accuracy.

For most of the wave theories that have been comprehensively presented and tabulated for general use, ϵ^* is at least 2×10^{-3}, as has been demonstrated by Dean (1970) in a comparative study of numerous theories available at that time. Dean's comparisons included his own stream-function theory (Dean 1965) taken to fifth order, the Stokes third and fifth order, cnoidal first and second order, Airy, and McCowan theories. Similar values of ϵ^* were deduced by von Schwind & Reid (1972) from an analysis of their own results and those of Chappelear (1961). Although these theories do not in general cover waves nearing maximum amplitude, von Schwind & Reid have taken some solutions for high waves to 60th order while Dean (1968) has taken his stream-function theory to 34th order.

Table 1. Computed coefficients

l	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.60
N	79	79	71	63	55	47	41	35
s	364217989	352381399	340907667	329816326	319102417	308758012	298779819	279906887
q	107984649	105703210	102973956	100088228	97102473	94035830	90961597	85042496
a_0	−24944881	−23600603	−22354523	−21204903	−20149440	−19185197	−18308629	−16801689
a_1	−421112255	−395974465	−374491303	−355610922	−338534695	−322778010	−308159387	−281991761
	70466660	59177638	51058634	44970235	40185980	36272222	33000427	27867836
	7993371	6477300	5510759	4823279	4283138	3830607	3444268	2832856
	1888981	1619113	1430926	1272536	1129876	1000645	886993	708301
a_5	630219	601503	549296	488081	427285	371400	323017	249953
	276794	287056	262175	228473	195546	166318	141872	106619
	150438	159453	142058	120420	100604	83789	70192	51364
	93931	96957	83366	68670	56084	45819	37745	26941
	63520	62400	51696	41462	33179	26630	21591	15041
a_{10}	44923	41755	33410	26164	20556	16227	12953	8804
	32635	28769	22312	17107	13216	10267	8067	5346
	24139	20292	15310	11517	8759	6698	5178	3340
	18099	14598	10750	7948	5955	4481	3405	2135
	13725	10682	7698	5601	4135	3061	2284	1388
a_{15}	10512	7934	5608	4019	2924	2127	1557	915
	8123	5973	4147	2929	2100	1500	1075	609
	6330	4550	3107	2164	1528	1071	751	408
	4971	3504	2356	1617	1125	773	528	274
	3932	2724	1805	1222	836	562	374	184
a_{20}	3131	2137	1395	931	627	412	266	124
	2508	1689	1088	715	473	303	189	83
	2021	1345	854	554	360	224	135	55
	1638	1078	675	431	275	166	96	36
	1333	869	537	337	211	123	68	23
a_{25}	1090	704	429	265	162	92	48	14
	895	574	345	209	125	68	34	9
	738	469	278	166	97	50	23	5
	610	385	225	132	75	37	16	3
	506	318	182	105	58	27	11	1
a_{30}	421	262	148	84	45	20	7	1
	351	218	121	67	35	15	5	
	294	181	99	53	27	10	3	
	246	151	81	43	21	7	2	
	206	126	66	34	16	5	1	
a_{35}	173	105	55	27	12	4		
	146	88	45	22	9	2		
	123	74	37	17	7	2		
	104	62	30	14	5	1		
	87	52	25	11	4	1		
a_{40}	74	44	20	9	3			
	62	37	17	7	2			
	52	31	14	5	1			
	44	26	11	4	1			
	37	22	9	3	1			
$^{8}a_{45}$	31	18	7	2				
	26	15	6	2				
	22	13	5	1				
	18	11	4	1				
	15	9	3	1				
$^{8}a_{50}$	13	7	2	1				
	11	6	2					
	9	5	2					
	7	4	1					
	6	3	1					
$^{8}a_{55}$	5	3	1					
	4	2	1					
	3	2						
	3	1						
	2	1						
$^{8}a_{60}$	2	1						
	1	1						
	1	1						
	1							
	1							

Table 1 (*continued*)

d	0.70	0.80	0.90	1.0	1.2	1.4	1.6
N	32	29	27	26	23	22	21
^{3}s	262 419 199	246 240 469	231 307 703	217 548 875	193 226 974	172 679 889	155 313 213
^{3}q	79 432 418	74 076 996	69 108 466	64 567 560	56 486 917	49 902 836	44 453 570
$^{3}a_0$	−15 591 015	−14 634 687	−13 889 755	−13 315 694	−12 542 598	−12 095 518	−11 835 880
$^{3}a_1$	−259 073 223	−238 675 835	−220 602 786	−204 605 126	−177 458 000	−155 928 937	−138 515 834
	23 981 354	20 889 547	18 411 587	16 408 701	13 334 238	11 200 686	9 633 651
	2 373 383	2 014 526	1 736 339	1 519 860	1 198 373	988 269	838 906
	578 424	480 145	406 558	351 168	270 388	219 898	184 635
$^{3}a_5$	199 445	162 369	135 394	115 638	87 046	69 868	58 017
	83 356	66 661	54 789	46 301	34 037	26 954	22 115
	39 415	30 974	25 089	20 977	15 026	11 729	9 495
	20 305	15 672	12 500	10 336	7 191	5 525	4 404
	11 133	8 430	6 613	5 404	3 636	2 745	2 149
$^{3}a_{10}$	6 396	4 744	3 654	2 948	1 908	1 412	1 082
	3 808	2 761	2 083	1 657	1 025	741	553
	2 330	1 647	1 214	951	557	391	283
	1 455	999	718	552	303	206	143
	923	614	428	322	163	106	70
$^{3}a_{15}$	591	379	255	187	86	53	32
	382	234	151	108	43	25	14
	247	144	88	61	21	11	5
	160	88	50	33	9	4	1
	103	53	28	17	3	1	
$^{3}a_{20}$	65	31	15	8	1		
	41	17	7	4			
	25	9	3	1			
	15	5	1				
	9	2					
$^{3}a_{25}$	5	1					
	2						
	1						

d	1.8	2.0	2.5	3.5	5.0	7.0	10.0
N	20	20	19	19	19	19	19
^{9}s	1 406 013 064	1 280 944 490	1 040 815 016	749 272 049	525 099 376	375 093 364	262 565 646
^{9}q	399 289 291	362 210 787	291 969 452	209 528 797	146 770 375	104 839 653	73 387 803
$^{9}a_0$	−116 826 503	−115 900 483	−114 842 327	−114 367 230	−114 300 051	−114 296 620	−114 296 556
$^{9}a_1$	−1 242 646 421	−1 125 738 462	−906 986 555	−650 418 369	−455 557 217	−325 407 704	−227 785 516
	84 443 147	75 406 450	59 468 990	42 236 354	29 539 778	21 098 895	14 769 214
	7 280 908	6 472 465	5 057 763	3 582 081	2 504 235	1 788 621	1 252 033
	1 587 810	1 406 689	1 088 985	769 484	537 760	384 082	268 857
$^{9}a_5$	493 973	436 305	334 204	235 668	164 647	117 593	82 315
	186 156	163 969	124 042	87 306	60 978	43 551	30 485
	78 863	69 285	51 632	36 278	25 332	18 092	12 664
	36 003	31 555	23 085	16 195	11 305	8 074	5 652
	17 229	15 067	10 772	7 546	5 266	3 761	2 633
$^{9}a_{10}$	8 466	7 389	5 130	3 589	2 504	1 788	1 252
	4 197	3 657	2 444	1 708	1 191	851	596
	2 064	1 796	1 140	796	555	397	278
	988	858	508	355	247	177	124
	449	390	209	145	101	72	51
$^{9}a_{15}$	187	162	74	52	36	26	18
	68	59	21	14	10	7	5
	19	16	3	2	2	1	1
	3	3					

All of the above methods, although differing widely in detail, fall essentially into category (A) of § 2. The solutions consist of a finite series of terms, without specific allowance for the form of the crest of the maximum wave. To illustrate the relative accuracy to be expected from maximum-amplitude solutions of forms (A), (B) and (C) the present solutions have been expressed as infinite series according to (6.1) and (6.2). Table 3 summarizes the contribution made to each total coefficient A_m by its components s_m, q_m and a_m; the tabulated values show, for j ranging from 3 to 7, the lowest value of m for which the relevant coefficient is less than 0.5×10^{-j}. Each set of three tabulated values thus gives an indication, for methods of type (A), (B) and (C) respectively, of the number of terms needed to achieve a solution of broadly j-decimal accuracy.

Table 2. Error quantities derived from p_s for the solutions given in table 1

d	$10^6\hat{p}_s$	$10^6\epsilon^*$	d	$10^6\hat{p}_s$	$10^6\epsilon^*$
0.20	3	0.2	1.0	10	0.6
0.25	2	0.2	1.2	10	1.1
0.30	4	0.4	1.4	11	1.1
0.35	5	0.5	1.6	11	1.2
0.40	6	0.6	1.8	11	1.6
0.45	6	0.1	2.0	12	1.4
0.5	7	0.3	2.5	12	1.8
0.6	7	0.4	3.5	12	1.6
0.7	8	0.2	5.0	12	1.6
0.8	9	0.6	7.0	12	1.6
0.9	9	0.7	10.0	12	1.6

Table 3. Magnitudes of s_m, q_m, a_m (defined in (6.1)) in selected solutions

(The table shows the value of m for which the relevant coefficient becomes less than 0.5×10^{-j}.)

d ...	0.2			0.5			1.0			2.0			10.0		
j	s	q	a	s	q	a	s	q	a	s	q	a	s	q	a
3	141	32	10	97	20	8	71	15	6	49	11	5	19	7	4
4	560	78	18	383	50	13	280	36	10	194	27	8	75	15	6
5	2200	197	30	1530	125	19	1110	90	14	770	67	11	296	35	9
6	8900	498	43	6100	315	26	4400	228	18	3100	168	14	1180	88	12
7	35300	1270	55	24200	800	31	17700	580	21	12200	425	17	4700	221	15

It is at once clear from table 3 that the present solutions of about six-decimal accuracy could have been achieved under method (A), if at all, only with the use of several thousand terms. With method (B) several hundred terms would still have been needed except perhaps at the deep-water extreme. The high accuracy achieved in the present work from a relatively modest number of terms is undoubtedly attributable to the second crest term in method (C) which does not appear to have been used before in a systematic series of solutions.

Method (B) has been used in the past for several solutions of relatively low order for maximum waves. These include the solitary wave solutions of Yamada (1957*b*), who obtained $\epsilon^* \approx 7 \times 10^{-3}$ from thirteen terms, and Lenau (1966), who obtained $\epsilon^* \approx 2 \times 10^{-3}$ from nine terms. Yamada (1957*a*) has also computed the deep-water wave to get $\epsilon^* \approx 4 \times 10^{-4}$ from thirteen terms. These values are reasonably consistent with table 3 although the solitary wave solutions are more accurate than might have been expected.

The only solutions for maximum waves whose accuracy can be expected to be comparable with the present work are those of Schwartz (1974) and Cokelet (1977) who both used a computer-aided expansion process taken to high order. Their formulations for general amplitudes are essentially of type (A) and to reach maximum amplitude they used an element of extrapolation or implied extrapolation. Schwartz extrapolated for the height of the maximum wave and built this assumed height into a formulation of type (B). Cokelet converted his computed finite series to a corresponding infinite series by using its Padé approximant. For deducing the expected accuracy from table 3 this may also be regarded as a solution of type (B).

Table 4. Height : wavelength ratio for maximum waves. Comparison of present results with those of Schwartz (1974) and Cokelet (1977)

$\exp(-d)$	Schwartz (1974)	Cokelet (1977)	this paper
0	0.14118	0.141055	0.141063
0.1	0.1380	0.1378	0.137801
0.2	0.1285	0.1285	0.128495
0.3	0.1145	0.11443	0.114439
0.4	0.0975	0.09739	0.097374
0.5	0.0791	0.07910	0.079072
0.6	0.0614	0.06090	0.060984
0.7	0.045	0.04374	0.043975
0.8	—	0.0279	0.028258
0.9	—	0.015	0.013667

Schwartz worked generally to order 48 and, for the deep-water wave only, to order 117 while Cokelet worked throughout to order 110. Referring to table 3, we should *a priori* expect both sets of results for the maximum deep-water wave to be of comparable accuracy with the present work except that the latter might have an advantage in being free from extrapolation. At shallower depths we should expect the present results to show progressively greater accuracy than either Schwartz or Cokelet. To get a specific comparison the present program was rerun for the cases covered by them; values of the height: wavelength ratio H/L obtained from the three methods are shown in table 4. The last case, $d = -\ln 0.9 = 0.105$, was not computed specifically but was deduced from the solution for $d = 0.2$. For the maximum deep-water wave the present result for H/L agrees with Cokelet's to about five decimal places. Schwartz's result, being in effect the extrapolated value he needed to construct his solution, appears to be less accurate. At shallower depths a progressively greater discrepancy develops, as already suggested.

The comparisons of this section suggest strongly that the present results show an important improvement in accuracy over previous work. This view will be consolidated in the next two sections, which will compare the results with previous studies made at the two ends of the range, the solitary wave and deep-water wave respectively.

9. The solitary wave

The extreme case of the solitary wave is provided in our formulation by $\lambda = \infty$, $R = 1$ and cannot be computed specifically because the computing domain degenerates to the circumference of the unit circle. However, when $d = 0.2$ the computed solution shows a velocity profile at the trough which is uniform between surface and bed to almost seven-decimal accuracy; this may therefore be regarded as a valid solution for any greater wavelength, including the maximum solitary wave. Although solutions are obtainable down to $d = 0.1$, beyond which the iteration fails because α and a_0 begin to lose their independence, the accuracy attainable for a given N falls away because most of the nodal points are situated on the uniform part of the profile and do not contribute to defining the crest shape. We have therefore used $d = 0.2$ as the lower limit of the results presented in table 1 and have deduced the solitary wave solution from it, using the relations derived in the latter part of § 4.

Table 1 gives the computed solution for $d = 0.2$, $N = 79$, $\theta_c = \frac{1}{280}\pi$, for which $\hat{p}_s$, the maximum modulus of p_s, is 2.7×10^{-6}. By increasing N, $\hat{p}_s$ could be reduced further but the computing requirements would become excessive before a significant reduction were achieved. Nevertheless, there is a strong incentive to determine the best possible solution for the maximum solitary wave since the shallow-water end of the range poses the greatest problems for most methods. An attempt has therefore been made to extrapolate some of the results to their limiting values for zero $\hat{p}_s$.

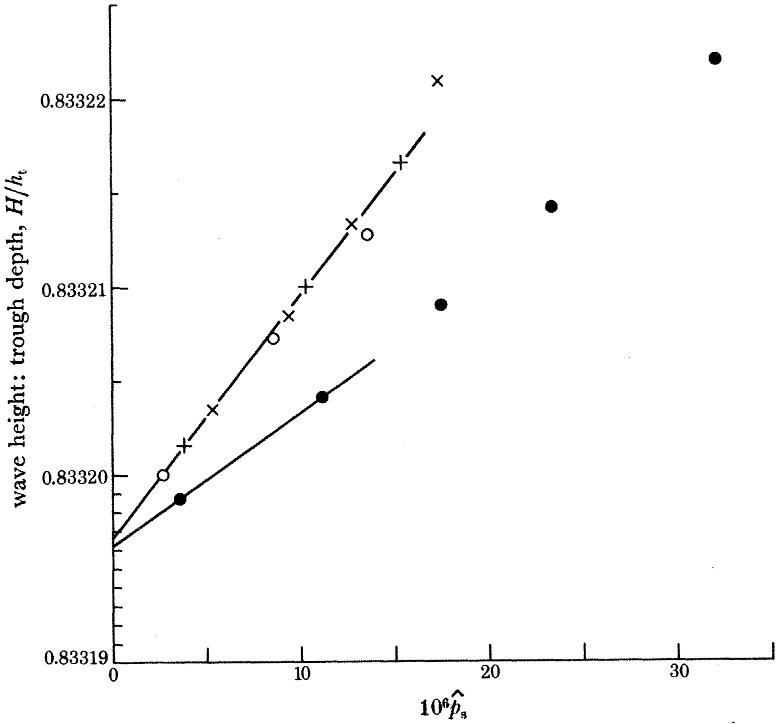

FIGURE 4. The variation of $\hat{p}_s$ with height: trough depth ratio H/h_t for shallow-water solutions; the extrapolation to $\hat{p}_s = 0$ gives an improved estimate for H', the height: depth ratio of the maximum solitary wave.

	d	θ_c	N
●	0.20	$\frac{1}{420}\pi$	79, 63, 57, 53, 49
○	0.20	$\frac{1}{280}\pi$	79, 63, 57
+	0.19	$\frac{1}{280}\pi$	79, 63, 57
×	0.18	$\frac{1}{280}\pi$	79, 69, 63, 57

A parameter of prime interest is the ratio of limiting height to undisturbed depth, $H/h_\infty = H'$. For our long periodic wave, with $d = 0.2$, h_∞ is approximated by the trough depth h_t so that

$$H' = H/h_\infty \approx H/h_t. \tag{9.1}$$

It was found from a series of computations for $d = 0.2$ at increasing values of N up to 79 that H/h_t varied almost linearly with $\hat{p}_s$. Figure 4 shows the results from two such series with $\theta_c = \frac{1}{420}\pi$

and $\frac{1}{280}\pi$ respectively. As N increases the results approach straight lines of different slope but with an almost identical intercept at $\hat{p}_s = 0$.

As an additional check that $d = 0.20$ gives a valid representation of the solitary wave two further sets of results for $d = 0.19$ and 0.18, with $\theta_c = \frac{1}{280}\pi$, are included in figure 4. All three sets for $\theta_c = \frac{1}{280}\pi$ are seen to converge to a common line with an intercept at $\hat{p}_s = 0$ slightly below $H/h_t = 0.833\,197$. The results for $\theta_c = \frac{1}{420}\pi$ show an intercept which is a little lower, but greater than $H/h_t = 0.833\,196$. From the results summarized in figure 4 we thus deduce that for the maximum solitary wave $H' = 0.833\,197$ with an uncertainty of about one unit in the sixth decimal place.

Table 5. Properties of the maximum solitary wave and comparison with previous results

(Values are normalized such that gravity and undisturbed depth are each taken as unity.)

	Longuet-Higgins & Fenton (1974)	Fox (1977)	this paper: computed from $d = 0.2$	this paper: extrapolated to $\hat{p}_s = 0$
height/undisturbed depth, H'	0.827	0.8332	0.833200	0.833197
Froude number	1.286	1.2909	1.290891	1.290889
M_∞	1.897	1.968	1.970323	1.970319
C_∞	1.653	1.713	1.714571	1.714569
I_∞	2.440	2.540	2.543474	2.543463
T_∞	0.5052	0.5339	0.535012	0.535005
V_∞	0.413	0.4369	0.437675	0.437670

The integral properties of the maximum solitary wave have been calculated from the solution for $d = 0.2$ in table 1, according to the method set out in § 4, and reduced to Longuet-Higgins's normalized form. The left-hand sides of (4.26)–(4.28) (with $h_\infty = 2F^2 = 1$) all have a modulus not exceeding 2×10^{-6}, thus confirming again that the solution is, to the above-specified accuracy, a valid representation of the solitary wave. The integral values, given with H' in the third column of table 5, have also been extrapolated from a series of runs to their estimated true values in the manner described for H'. Additional guidance in the extrapolation process was obtained from the need for the final values to satisfy (4.26)–(4.28). The extrapolated values are given in the fourth column of table 5.

For comparison with these values we have predictions by Longuet-Higgins & Fenton (1974) and Fox (1977) which are shown in the first and second columns of table 5. Our results agree with those of Fox to between three and five significant figures but the discrepancy with Longuet-Higgins & Fenton's results is greater. Both methods used series-expansion techniques; Fox assumed the form of the maximum crest and Longuet-Higgins & Fenton used Padé approximants to generate the infinite series needed for the maximum wave.

A further independent estimate of H' is provided by Witting (1975) who has provisionally reported $H' = 0.8332$ from an integral equation solution based on the method of Yamada (1957 *b*).

Byatt-Smith & Longuet-Higgins (1976) have developed an integral equation method for steep solitary waves somewhat short of the highest. For the highest waves computed by them they also found a discrepancy with the results obtained from Padé approximants by Longuet-Higgins & Fenton (1974). The present result for the Froude number of the highest solitary wave, 1.2909, if plotted on figure 1 of their paper, is seen to be consistent with their integral equation results.

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To summarize, the present computation of the maximum solitary wave has been shown to have high inherent accuracy, demonstrated by the small values of $\hat{p}_S$, P_1, P_2, P_3 and its consistency with equations (4.26)–(4.28). It is also consistent with several previous accurate computations of maximum and near-maximum waves, with the single exception of the results of Longuet-Higgins & Fenton (1974), based on Padé approximants. It is therefore believed to be an authentic solution whose accuracy, verging on six decimal places, has not previously been achieved.

10. The deep-water wave

The infinite-depth case corresponds to $d = \infty$, $R = 0$ in our formulation but again cannot be computed directly because the wave height tends to zero as a consequence of distances being normalized with respect to a vertical dimension. However, suppose that d is large enough for R^2 to be neglected, to computing accuracy. The terms involving R^2/τ in (2.15) and (3.7) will then vanish, except at depths sufficient to reduce τ to order R, to make ζ_m and $\zeta_{m,A,\nu}$ sensibly independent of R. Since also $R = \exp(-d)$, it follows from (2.5) that F^2 may be taken as $1/d$, which allows the free-surface condition (2.13) to be written in the form

$$(\alpha+\eta d)\left\{\left[1-\frac{1}{2}\frac{\partial(\eta d)}{\partial\rho}\right]^2+\left[\frac{1}{2}\frac{\partial(\eta d)}{\partial\theta}\right]^2\right\}\approx 1, \quad \rho = 1. \tag{10.1}$$

Table 6. Properties of the maximum deep-water wave and comparison with previous results

(Values are normalized such that gravity = 1 and wavelength = 2π.)

	Longuet-Higgins (1975)	Cokelet (1977)	Longuet-Higgins & Fox (1978)	this paper
height/wavelength	0.1411	0.141055	0.14107	0.141063
phase velocity	1.0923	1.0922	1.0923	1.092282
I	0.0701	0.0701	0.07011	0.070113
T	0.0383	0.03827	0.03829	0.038292
V	0.0346	0.03457	0.03457	0.034568

This implies that, at the surface, the function $\alpha+\eta d$ is now independent of d. Within the range of d for which F^2 and R satisfy the above conditions the coefficients of the solution are thus constrained by

$$\left.\begin{aligned} \alpha - a_0 d &\rightarrow \text{constant},\\ a_0 &\rightarrow \text{constant},\\ a_m d &\rightarrow \text{constant}, \quad m = 1,2,\ldots,N-2. \end{aligned}\right\} \tag{10.2}$$

We have computed solutions as far as $d = 10$, for which $R^2 = 2\times10^{-9}$ and F^2 differs from $1/d$ by only 4×10^{-10}. We may therefore regard the coefficients of this solution as applicable also to any greater depth, including the infinite-depth wave.

The integral parameters computed for $d = 10$ may be compared with predictions for the infinite-depth maximum wave by Longuet-Higgins (1975), Cokelet (1977) and Longuet-Higgins & Fox (1978). As with the solitary wave these were all derived from series-expansion methods covering a range of wave amplitudes. Results for the maximum wave were obtained in the first two papers by the use of Padé approximants and in the third by an assumed form for the crest.

The present results have been renormalized according to Longuet-Higgins's scheme, with wavelength rescaled to 2π and gravity to unity. All four sets are given in table 6; there is very good agreement throughout with evidence of progressively increasing accuracy. The only minor inconsistency concerns the height: wavelength ratio, for which Cokelet's six-decimal estimate differs from ours by eight units in the last place. Table 2 shows that our solution gives $\hat{p}_s = 12\times 10^{-6}$, which implies a corresponding surface displacement error of 2.5×10^{-6} and an error in the height: wavelength ratio of order 2×10^{-6}. Although it is not possible to extrapolate to zero $\hat{p}_s$ by varying N, as was done with the solitary wave solutions, it would be surprising if our estimate of 0.141 063 were in error by significantly more than two units in the last place. Similarly the errors in the integral identities (4.17) and (4.18) (about 2×10^{-6} in normalized form) are small enough to suggest that the six-decimal results quoted in table 6 are also accurate to within one or two units in the last place.†

Angular momentum

In a recent paper Longuet-Higgins (1980) discusses the angular momentum of gravity waves and defines the 'level of action' of a wavetrain as the level about which the long-term Langrangian-mean angular momentum vanishes. For the deep-water wave Longuet-Higgins has calculated the level of action over the full range of amplitudes. At maximum amplitude he shows that the level is very close to the crest and raises the question of whether a more accurate calculation would show it to be exactly at the crest. The present results provide an opportunity of repeating the calculation in the hope of improving the accuracy. The infinite series of Fourier coefficients A_m, $m = 1, 2, \ldots, \infty$, of our solution can be obtained from (6.2) and the computed coefficients given in table 1. The integral parameters are available from table 6. We can therefore repeat the numerical procedure described in § 9 of Longuet-Higgins's paper and derive revised values for the last line of his tables 1 and 2.

There is no difficulty in incorporating as many terms as required in the summations to get results to, apparently, high accuracy. Thus, in Longuet-Higgins's notation (in which y is taken as positive upwards relative to mean level, wavelength is scaled to 2π and gravity to unity):

$$\begin{aligned} y_{\max} &= 0.596\,541 \text{ (the level of the crest)},\\ \bar{A}_{\mathrm{E}} &= 0.008\,135 \text{ (Eulerian-mean angular momentum)},\\ \bar{A}_{\mathrm{L}} &= 0.042\,612 \text{ (long-term Lagrangian-mean angular momentum)},\\ y_{\mathrm{a}} &= 0.60\,777 \text{ (the level of action)}. \end{aligned}$$

It must be remembered, however, that the coefficients A_m have incorporated a coefficient s that is too high by 0.19 %, as pointed out in § 6 and Appendix 2. It is difficult to be sure of the extent to which this will influence the results; some experimental computations on trial perturbations of the coefficients suggest that the corrections to be applied to the last five of the above quantities are of order $+0.0005$, -0.0003, $+0.0004$, -0.0001, -0.0012 respectively. Consequently, y_{a} can probably be specified to three decimals as 0.607. This result indicates that the level of action is apparently slightly above the level of the crest, $y_{\max}$, and not below it as suggested by Longuet-Higgins's first calculation.

† The expected accuracy is corroborated by the recent work of Olfe & Rottman (1980); their monotonically increasing sequences of estimates for the quantities of table 6 have final values below ours by 4×10^{-6} or less. Their results were obtained in the course of calculating classes of limiting *non-uniform* wavetrains, in which not all crests have the same form.

11. Detailed computation of the wave motion

The coefficients given in table 1 are sufficient to determine any required property of the flow. This section summarizes the formulae needed to do this and continues with detailed tabulations of sufficient cases to make the results of the work readily available for application and further study. The flow is throughout regarded as being in its steady state with a stationary wave profile. A relevant phase velocity may then be superimposed as required.

The computed coefficients for scheme (C) together with the definitions of $\zeta_{m,A,\nu}$, equation (3.7), and z_0, equation (2.10), define $z = z_0 + \zeta$ as a function of $\chi = \mathrm{i}(2/d)\ln\tau$. Velocity is then given by the complex conjugate of

$$1/(1+\tfrac{1}{2}\mathrm{i}d\tau\,\mathrm{d}\zeta/\mathrm{d}\tau), \tag{11.1}$$

and acceleration by the complex conjugate of

$$\tfrac{1}{4}d^2\left(\tau\frac{\mathrm{d}\zeta}{\mathrm{d}\tau}+\tau^2\frac{\mathrm{d}^2\zeta}{\mathrm{d}\tau^2}\right)\Big/\left|1+\tfrac{1}{2}\mathrm{i}d\tau\frac{\mathrm{d}\zeta}{\mathrm{d}\tau}\right|^2\left(1+\tfrac{1}{2}\mathrm{i}d\tau\frac{\mathrm{d}\zeta}{\mathrm{d}\tau}\right)^2. \tag{11.2}$$

Partial derivatives of ξ, η may be obtained from ζ according to the relations

$$\left.\begin{aligned}\tau\frac{\mathrm{d}\zeta}{\mathrm{d}\tau} &= \frac{\partial\eta}{\partial\theta}-\mathrm{i}\frac{\partial\xi}{\partial\theta} = \rho\frac{\partial\xi}{\partial\rho}+\mathrm{i}\rho\frac{\partial\eta}{\partial\rho},\\ \tau^2\frac{\mathrm{d}^2\zeta}{\mathrm{d}\tau^2} &= \rho^2\frac{\partial^2\xi}{\partial\rho^2}+\mathrm{i}\rho^2\frac{\partial^2\eta}{\partial\rho^2},\\ \tau^2\frac{\mathrm{d}^2\zeta}{\mathrm{d}\tau^2}+\tau\frac{\mathrm{d}\zeta}{\mathrm{d}\tau} &= -\frac{\partial^2\xi}{\partial\theta^2}-\mathrm{i}\frac{\partial^2\eta}{\partial\theta^2} = \rho\frac{\partial^2\eta}{\partial\rho\,\partial\theta}-\mathrm{i}\rho\frac{\partial^2\xi}{\partial\rho\,\partial\theta}.\end{aligned}\right\} \tag{11.3}$$

The pressure may be deduced from Bernoulli's equation as

$$p = \tfrac{1}{2}[y/F^2+\alpha-1-u^2-v^2]. \tag{11.4}$$

An important field variable is $t(\chi)$, defined as the time taken by a particle in a given position to travel from a starting point beneath the wave crest. Particles aligned on any vertical section in the steady flow will in general have different travel times from the wave crest and it is this fact that introduces mass-transport effects when a celerity is superimposed. The time t is given by

$$t(\rho,\theta) = (2/d)\int_0^\theta |1+\tfrac{1}{2}\mathrm{i}d\tau\,\mathrm{d}\zeta/\mathrm{d}\tau|^2\,\mathrm{d}\theta, \tag{11.5}$$

the integral being evaluated along a streamline, ρ = constant. For a small-amplitude wave, with components of the form ζ_m, t may be found by summation, by exploiting the orthogonal properties of $\cos m\theta$, $\sin m\theta$, but for the present maximum waves a numerical quadrature is needed, which makes t the most difficult field variable to compute. For the important portion of the surface streamline near the crest a ten-term series expansion in powers of θ is needed to ensure sufficient accuracy; the details of this expansion are set out in Appendix 2. This Appendix also evaluates the error in t arising from the incorrect crest acceleration.

Of the overall wave properties the wavelength is given simply by

$$L = \lambda(1+\tfrac{1}{2}a_0), \tag{11.6}$$

while the mean depth is best found from (4.19) in the form

$$h = 2-F^2(\sigma_b^2+2P_1-\alpha+c^2). \tag{11.7}$$

The quantities σ_b and P_1 must be found by quadrature but, being small, they can be found to the required accuracy with a relatively coarse interval compared with that needed for a direct quadrature for $\int y_s \, dx$. For similar reasons S_{xx} is computed from (4.16) rather than from its original definition, (4.8).

The celerity $c = \lambda/L$, already defined, ensures that the space-mean bed velocity over a wavelength will be zero. A second important celerity is

$$c' = 2/h, \tag{11.8}$$

at which speed of propagation the net mass transport passing any vertical section during a wave period will be zero.

Table 7 gives, for each of the 22 cases covered by table 1, a list of the overall properties of the wave motion. Detailed tables of field variables to four decimal places are given for five of these cases in tables 8–12. Table 8 gives the solution for $d = 0.2$; this is also applicable to any greater wavelength, including the solitary wave, if the appropriate length of uniform flow is added at the trough and the obvious consequential changes are made. Tables 9–11 apply to $d = 0.5$, 1.0, 2.0 respectively. Table 12, for $d = 10.0$, is also applicable to any greater depth, including the limiting deep-water wave, provided the small corrections indicated are made for $\psi = -2$, to remove the influence of the bed. For $\psi < -2$ the deep-water wave is given, to the accuracy of table 12, by a uniform flow.

Tables 8–12 give, besides a general presentation of the waveform, a more detailed coverage of the streamlines near to the surface. This allows the strong gradient of drift velocity found in a maximum wave to be properly defined, as will be shown in the next section.

To facilitate the rescaling of the tables to alternative normalizing systems each set of results shows the relevant value of gravity $(=1/2F^2)$ and of key dimensions of the waveform.

It is necessary to clarify the entries for acceleration at the crest in tables 8–12. As shown by Longuet-Higgins & Fox (1977), the acceleration in the Stokes corner flow is $\frac{1}{2}g$ directed always away from the crest so that at the crest itself the acceleration is not unique but takes a limiting value dependent on the path by which the crest is approached. We have tabulated the limiting values applicable to the vertical beneath the crest ($\phi = 0$), the horizontal component then being zero. The vertical component, if multiplied by $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$, gives respectively the horizontal and vertical limiting accelerations if the crest is approached along the surface ($\psi = 0$). These computed crest accelerations all differ from the correct values for the Stokes corner flow on account of the method used, as discussed in § 6 and Appendix 2. The correct limiting vertical acceleration on $\phi = 0$ is $\frac{1}{2}g = 1/4F^2$, with the remaining limits in proportion.

12. Particle paths and drift profiles

To illustrate the use of the results to determine details of the wave motion we now calculate some specimen particle paths and drift profiles. At the same time we shall compare the results with some recent estimates by Longuet-Higgins (1979) based on very simple approximations for the solitary wave and the deep-water wave.

Particle paths in the steady motion follow from successive sets of x, y, t coordinates selected from the tables. Any general celerity $\tilde{c}$ may be superimposed, in which case y and t remain unchanged but x must be replaced by $\tilde{c}t - x$. For the results to be presented we have chosen $\tilde{c} = c = \lambda/L$.

Table 7. Properties of the waves defined in table 1

d		0.20	0.25	0.30	0.35
gravity	$1/2F^2$	0.506649	0.510374	0.514911	0.520252
mean depth	h	1.78061	1.80021	1.81830	1.83491
wavelength	L	54.995	44.334	37.2060	32.0972
wave height	H	1.39940	1.39598	1.39183	1.38691
total head relative to bed	$2+\alpha F^2$	3.07894	3.07143	3.06238	3.05186
mean depth/wavelength	h/L	0.032378	0.040606	0.048871	0.057167
height/mean depth	H/h	0.78591	0.77545	0.76546	0.75585
celerity (zero mean velocity)	c	1.14250	1.13379	1.12584	1.11860
celerity (zero mass transport)	c'	1.12321	1.11098	1.09993	1.08997
depth parameter	$2h(Fc/L)^2$	0.001517	0.002307	0.003233	0.004284
height parameter	$2H(Fc/L)^2$	0.001192	0.001789	0.002475	0.003238
mean momentum	I	0.034339	0.041057	0.047109	0.052528
mean kinetic energy	T	0.019616	0.023275	0.026518	0.029379
mean potential energy	V	0.016516	0.019725	0.022614	0.025202
radiation stress	S_{xx}	0.047251	0.055735	0.063091	0.069401
mean energy flux	E	0.040145	0.047052	0.052963	0.057977
bed velocity variance	σ_b^2	0.010297	0.012115	0.013672	0.014983
trough velocity variance	σ_t^2	0.000000	0.000000	0.000000	0.000000

d	0.40	0.45	0.50	0.60	0.70	0.80
$1/2F^2$	0.526386	0.533303	0.540988	0.558608	0.579118	0.602376
h	1.85008	1.86384	1.87625	1.89725	1.91362	1.92595
L	28.2509	25.2465	22.8320	19.18448	16.55251	14.55856
H	1.38110	1.37424	1.36616	1.34574	1.31924	1.28696
$2+\alpha F^2$	3.03996	3.02676	3.01237	2.980386	2.944908	2.906870
h/L	0.065487	0.073826	0.082176	0.098895	0.115609	0.132290
H/h	0.74651	0.73732	0.72813	0.70931	0.68940	0.66822
c	1.11203	1.10610	1.10077	1.091713	1.084546	1.078951
c'	1.08104	1.07305	1.06596	1.05416	1.04514	1.03845
$2h(Fc/L)^2$	0.005446	0.006708	0.008061	0.010999	0.014186	0.017561
$2H(Fc/L)^2$	0.004065	0.004946	0.005870	0.007801	0.009780	0.011734
I	0.057348	0.061600	0.065316	0.071256	0.075405	0.078007
T	0.031886	0.034068	0.035949	0.038896	0.040890	0.042083
V	0.027506	0.029543	0.031327	0.034190	0.036200	0.037466
S_{xx}	0.074739	0.079172	0.082762	0.087648	0.089861	0.089885
E	0.062181	0.065651	0.068454	0.072294	0.074134	0.074372
σ_b^2	0.016060	0.016916	0.017560	0.018255	0.018238	0.017629
σ_t^2	0.000000	0.000000	0.000000	0.000002	0.000007	0.000023

d	0.90	1.0	1.2	1.4	1.6	1.8
$1/2F^2$	0.628230	0.656518	0.719723	0.790646	0.867991	0.950564
h	1.93490	1.94110	1.94746	1.94865	1.94709	1.94421
L	12.99295	11.72972	9.81525	8.43313	7.38919	6.57352
H	1.24980	1.20900	1.12138	1.03252	0.94779	0.86995
$2+\alpha F^2$	2.867212	2.826809	2.746673	2.670869	2.601706	2.539970
h/L	0.148920	0.165486	0.198411	0.231071	0.263505	0.29576
H/h	0.64593	0.62284	0.57582	0.52986	0.48677	0.44746
c	1.074632	1.071327	1.066909	1.06437	1.06290	1.06204
c'	1.03364	1.03034	1.02698	1.02635	1.02717	1.02870
$2h(Fc/L)^2$	0.021069	0.024664	0.031971	0.039261	0.046416	0.053388
$2H(Fc/L)^2$	0.013609	0.015362	0.018409	0.020803	0.022594	0.023889
I	0.079310	0.079558	0.077760	0.074089	0.069565	0.064820
T	0.042615	0.042616	0.041481	0.039429	0.036970	0.034421
V	0.038104	0.038227	0.037353	0.035569	0.033375	0.031081
S_{xx}	0.088216	0.085327	0.077494	0.068841	0.060690	0.053576
E	0.073379	0.071491	0.066142	0.060013	0.054043	0.048654
σ_b^2	0.016573	0.015219	0.012133	0.009151	0.006643	0.004699
σ_t^2	0.000057	0.000116	0.000322	0.000637	0.001024	0.001436

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TABLE 7 (*continued*)

d	2.0	2.5	3.5	5.0	7.0	10.0
$1/2F^2$	1.037315	1.266959	1.753194	2.500227	3.500006	5.000000
h	1.94082	1.93243	1.92010	1.90990	1.90300	1.89781
L	5.91907	4.73792	3.38508	2.36964	1.69260	1.18482
H	0.79997	0.65789	0.47649	0.334233	0.238764	0.167135
$2+\alpha F^2$	2.485532	2.37712	2.24099	2.13487	2.06370	2.01030
h/L	0.32789	0.40787	0.56723	0.80599	1.12430	1.60177
H/h	0.41218	0.34044	0.24816	0.17500	0.12547	0.088067
c	1.06152	1.06092	1.06065	1.060614	1.060612	1.060612
c'	1.03049	1.03496	1.04161	1.04717	1.05098	1.05385
$2h(Fc/L)^2$	0.060175	0.076477	0.10752	0.15303	0.21349	0.30415
$2H(Fc/L)^2$	0.024803	0.026036	0.026683	0.026781	0.026786	0.026786
I	0.060208	0.050157	0.036563	0.025672	0.018340	0.012838
T	0.031956	0.026606	0.019390	0.013614	0.0097258	0.0068081
V	0.028856	0.024023	0.017505	0.012290	0.0087799	0.0061460
S_{xx}	0.047593	0.036790	0.025381	0.017602	0.012563	0.0087944
E	0.043965	0.035031	0.024746	0.017256	0.012322	0.0086252
σ_b^2	0.003266	0.001259	0.000174	0.000009	0.000000	0.000000
σ_t^2	0.001835	0.002649	0.003393	0.003369	0.002891	0.002269

Figure 5 shows the particle paths calculated in this way for the five cases presented in detail in tables 8–12. The plots have been scaled to apply to wave motions of a common mean depth h. Half only of each particle orbit has been plotted, the remaining half being a mirror image. The total advance of a particle after a complete orbit is twice the horizontal distance between the two ends of the relevant semi-orbit.

For $d = 0.2$ a set of alternative orbits is plotted to show the particle movement in the solitary wave. These are obtained from the computed steady motion by superimposing the uniform trough velocity, rather than c, thereby bringing the extremes of the wave motion to rest. The particle motion is now entirely forward during the passage of the wave crest with no compensating return movement. The extremes of these trajectories are linked in figure 5 to give a curve relating the forward displacement of a particle to its depth in the undisturbed fluid.

Longuet-Higgins (1979) has recently estimated the path of a surface particle in a solitary wave using his own previously derived simple approximation to the wave profile (Longuet-Higgins 1974). His computed path is plotted for comparison in figure 5 and shows very good agreement. The present results give a trajectory whose length in relation to the undisturbed water depth is 4.246 compared with Longuet-Higgins's estimate of 4.229.

In the same paper Longuet-Higgins (1979) derived the path of a surface particle in the maximum deep-water wave using the computations of Yamada (1957*a*) and Schwartz (1974). This profile, although not plotted in figure 5, is indistinguishable from the present result.

An effective drift velocity for a fluid particle is given by the ratio of the distance moved forward to the time taken in completing one cycle of the wave motion. Thus for a particular celerity $\tilde{c}$ and a particular streamline, or value of ρ in the τ-plane, the drift velocity $U(\rho)$ is given by

$$U(\rho) = [\tilde{c}t(\rho, 2\pi) - L]/t(\rho, 2\pi). \qquad (12.1)$$

The drift profiles corresponding to $\tilde{c} = c$ are plotted as functions of mean depth for the same five cases in figure 5. They are scaled as before to a common total mean depth h and also to a common gravity $(1/2F^2)$. The deep-water case may also be compared with the profile obtained from a simple approach by Longuet-Higgins (1979) who transformed six successive maximum wave

TABLE 8a. DISPLACEMENT AND VELOCITY FOR $d = 0.2$

(ALSO APPLICABLE TO THE SOLITARY WAVE)

($1/2F^2 = 0.506649$, $h = 1.78061$, $L = 54.9952$, $\bar{y}_s = 1.20627$, $h_\infty = 1.67954$.)

ψ \ ϕ/λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
					horizontal displacement, x						
0.0	0.0000	3.6290	6.3682	9.0255	11.6674	14.3063	16.9447	19.5829	22.2212	24.8594	27.4976
−0.2	0.0000	3.6068	6.3639	9.0247	11.6673	14.3063	16.9447	19.5829	22.2212	24.8594	27.4976
−0.4	0.0000	3.5864	6.3600	9.0240	11.6671	14.3063	16.9447	19.5829	22.2212	24.8594	27.4976
−0.6	0.0000	3.5679	6.3564	9.0233	11.6670	14.3062	16.9447	19.5829	22.2212	24.8594	27.4976
−0.8	0.0000	3.5515	6.3533	9.0227	11.6669	14.3062	16.9447	19.5829	22.2212	24.8594	27.4976
−1.0	0.0000	3.5375	6.3505	9.0221	11.6668	14.3062	16.9447	19.5829	22.2212	24.8594	27.4976
−1.2	0.0000	3.5258	6.3482	9.0217	11.6667	14.3062	16.9447	19.5829	22.2212	24.8594	27.4976
−1.4	0.0000	3.5167	6.3464	9.0214	11.6666	14.3062	16.9447	19.5829	22.2212	24.8594	27.4976
−1.6	0.0000	3.5101	6.3451	9.0211	11.6666	14.3062	16.9447	19.5829	22.2212	24.8594	27.4976
−1.8	0.0000	3.5061	6.3443	9.0210	11.6665	14.3062	16.9446	19.5829	22.2212	24.8594	27.4976
−2.0	0.0000	3.5048	6.3440	9.0209	11.6665	14.3062	16.9446	19.5829	22.2212	24.8594	27.4976
					vertical displacement, y						
0.0	−0.0921	1.0900	1.2658	1.2994	1.3058	1.3070	1.3073	1.3073	1.3073	1.3073	1.3073
−0.2	0.4847	1.2725	1.4365	1.4679	1.4739	1.4750	1.4752	1.4753	1.4753	1.4753	1.4753
−0.4	0.8412	1.4572	1.6076	1.6364	1.6419	1.6430	1.6432	1.6432	1.6432	1.6432	1.6432
−0.6	1.1521	1.6440	1.7791	1.8051	1.8100	1.8110	1.8112	1.8112	1.8112	1.8112	1.8112
−0.8	1.4396	1.8325	1.9509	1.9738	1.9781	1.9790	1.9791	1.9791	1.9791	1.9792	1.9792
−1.0	1.7127	2.0226	2.1231	2.1425	2.1462	2.1469	2.1471	2.1471	2.1471	2.1471	2.1471
−1.2	1.9765	2.2139	2.2955	2.3113	2.3143	2.3149	2.3150	2.3151	2.3151	2.3151	2.3151
−1.4	2.2339	2.4063	2.4682	2.4802	2.4825	2.4829	2.4830	2.4830	2.4830	2.4830	2.4830
−1.6	2.4870	2.5994	2.6410	2.6491	2.6506	2.6509	2.6510	2.6510	2.6510	2.6510	2.6510
−1.8	2.7376	2.7930	2.8139	2.8180	2.8187	2.8189	2.8189	2.8189	2.8189	2.8189	2.8189
−2.0	2.9869	2.9869	2.9869	2.9869	2.9869	2.9869	2.9869	2.9869	2.9869	2.9869	2.9869
					horizontal velocity, u						
0.0	0.0000	1.0858	1.1726	1.1874	1.1902	1.1907	1.1908	1.1908	1.1908	1.1908	1.1908
−0.2	0.5045	1.0746	1.1697	1.1868	1.1900	1.1907	1.1908	1.1908	1.1908	1.1908	1.1908
−0.4	0.6101	1.0648	1.1669	1.1863	1.1899	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
−0.6	0.6732	1.0565	1.1645	1.1857	1.1898	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
−0.8	0.7161	1.0495	1.1623	1.1853	1.1897	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
−1.0	0.7467	1.0438	1.1605	1.1849	1.1897	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
−1.2	0.7687	1.0392	1.1589	1.1846	1.1896	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
−1.4	0.7844	1.0357	1.1577	1.1843	1.1896	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
−1.6	0.7949	1.0333	1.1569	1.1842	1.1895	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
−1.8	0.8009	1.0318	1.1563	1.1840	1.1895	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
−2.0	0.8029	1.0314	1.1562	1.1840	1.1895	1.1906	1.1908	1.1908	1.1908	1.1908	1.1908
					vertical velocity, v						
0.0	0.0000	0.1374	0.0301	0.0059	0.0011	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
−0.2	0.0000	0.1249	0.0280	0.0055	0.0011	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
−0.4	0.0000	0.1118	0.0256	0.0051	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
−0.6	0.0000	0.0982	0.0229	0.0045	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
−0.8	0.0000	0.0844	0.0201	0.0040	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
−1.0	0.0000	0.0705	0.0170	0.0034	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
−1.2	0.0000	0.0564	0.0138	0.0028	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
−1.4	0.0000	0.0423	0.0105	0.0021	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
−1.6	0.0000	0.0282	0.0070	0.0014	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
−1.8	0.0000	0.0141	0.0035	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

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Table 8*b*. Time, pressure and acceleration for $d = 0{\cdot}2$
(also applicable to the solitary wave)

λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
ψ						time, t					
0.0	0.0000	5.9206	8.3216	10.5699	12.7916	15.0082	17.2240	19.4395	21.6550	23.8705	26.0860
0.2	0.0000	4.5938	7.0249	9.2789	11.5017	13.7186	15.9344	18.1499	20.3654	22.5809	24.7964
0.4	0.0000	4.3622	6.8210	9.0803	11.3041	13.5212	15.7370	17.9525	20.1681	22.3836	24.5991
0.6	0.0000	4.2261	6.7099	8.9741	11.1988	13.4161	15.6319	17.8475	20.0630	22.2785	24.4940
0.8	0.0000	4.1329	6.6388	8.9073	11.1329	13.3503	15.5662	17.7817	19.9973	22.2128	24.4283
1.0	0.0000	4.0653	6.5901	8.8624	11.0886	13.3062	15.5221	17.7376	19.9532	22.1687	24.3842
1.2	0.0000	4.0153	6.5559	8.8312	11.0581	13.2758	15.4917	17.7073	19.9228	22.1383	24.3538
1.4	0.0000	3.9792	6.5320	8.8098	11.0371	13.2549	15.4708	17.6864	19.9019	22.1175	24.3330
1.6	0.0000	3.9545	6.5162	8.7958	11.0235	13.2413	15.4572	17.6728	19.8883	22.1038	24.3193
1.8	0.0000	3.9402	6.5072	8.7878	11.0157	13.2336	15.4495	17.6651	19.8806	22.0961	24.3116
2.0	0.0000	3.9355	6.5042	8.7852	11.0132	13.2311	15.4470	17.6626	19.8781	22.0936	24.3091
						pressure, p					
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.1650	0.1062	0.0900	0.0861	0.0853	0.0851	0.0851	0.0851	0.0851	0.0851	0.0851
0.4	0.2868	0.2118	0.1799	0.1721	0.1706	0.1703	0.1702	0.1702	0.1702	0.1702	0.1702
0.6	0.4037	0.3166	0.2697	0.2582	0.2558	0.2554	0.2553	0.2553	0.2553	0.2553	0.2553
0.8	0.5196	0.4208	0.3594	0.3442	0.3411	0.3405	0.3404	0.3404	0.3404	0.3404	0.3404
1.0	0.6356	0.5242	0.4488	0.4301	0.4264	0.4256	0.4255	0.4255	0.4255	0.4255	0.4255
1.2	0.7525	0.6268	0.5380	0.5160	0.5116	0.5108	0.5106	0.5106	0.5106	0.5106	0.5106
1.4	0.8708	0.7285	0.6269	0.6019	0.5969	0.5959	0.5957	0.5957	0.5957	0.5957	0.5957
1.6	0.9908	0.8294	0.7155	0.6877	0.6821	0.6810	0.6808	0.6808	0.6808	0.6808	0.6808
1.8	1.1129	0.9293	0.8037	0.7734	0.7673	0.7661	0.7659	0.7659	0.7658	0.7658	0.7658
2.0	1.2376	1.0281	0.8916	0.8590	0.8525	0.8512	0.8510	0.8509	0.8509	0.8509	0.8509
						horizontal acceleration					
0.0	0.0000	0.0732	0.0135	0.0025	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0000	0.0753	0.0155	0.0030	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.4	0.0000	0.0765	0.0172	0.0034	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.6	0.0000	0.0770	0.0187	0.0037	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.8	0.0000	0.0770	0.0200	0.0041	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1.0	0.0000	0.0767	0.0211	0.0043	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1.2	0.0000	0.0763	0.0220	0.0046	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1.4	0.0000	0.0758	0.0227	0.0047	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1.6	0.0000	0.0754	0.0232	0.0049	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1.8	0.0000	0.0751	0.0235	0.0050	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
2.0	0.0000	0.0750	0.0236	0.0050	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
						vertical acceleration					
0.0	0.2531	−0.0715	−0.0211	−0.0044	−0.0008	−0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.1854	−0.0612	−0.0194	−0.0041	−0.0008	−0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
0.4	0.1456	−0.0517	−0.0176	−0.0037	−0.0007	−0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.6	0.1158	−0.0431	−0.0156	−0.0033	−0.0007	−0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.8	0.0920	−0.0352	−0.0136	−0.0029	−0.0006	−0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1.0	0.0721	−0.0282	−0.0114	−0.0025	−0.0005	−0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1.2	0.0550	−0.0217	−0.0092	−0.0020	−0.0004	−0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1.4	0.0398	−0.0158	−0.0070	−0.0015	−0.0003	−0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1.6	0.0258	−0.0103	−0.0047	−0.0010	−0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.8	0.0127	−0.0051	−0.0023	−0.0005	−0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

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Table 8c. Displacement and velocity near the surface for $d = 0.2$
(also applicable to the solitary wave)

/λ ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ						horizontal displacement, *x*						
0.00	0.0000	0.0485	0.0771	0.1424	0.2269	0.2983	0.4219	0.6785	1.1011	1.6447	2.1380	27.4976
0.01	0.0000	0.0313	0.0593	0.1270	0.2142	0.2872	0.4126	0.6714	1.0961	1.6411	2.1353	27.4976
0.02	0.0000	0.0254	0.0499	0.1149	0.2028	0.2768	0.4036	0.6645	1.0910	1.6375	2.1326	27.4976
0.03	0.0000	0.0224	0.0443	0.1057	0.1927	0.2671	0.3951	0.6577	1.0860	1.6339	2.1299	27.4976
0.04	0.0000	0.0204	0.0406	0.0985	0.1839	0.2583	0.3869	0.6510	1.0811	1.6304	2.1272	27.4976
0.05	0.0000	0.0190	0.0379	0.0928	0.1762	0.2501	0.3791	0.6445	1.0762	1.6269	2.1246	27.4976
0.06	0.0000	0.0180	0.0358	0.0882	0.1695	0.2427	0.3716	0.6382	1.0714	1.6234	2.1220	27.4976
0.07	0.0000	0.0171	0.0342	0.0844	0.1635	0.2359	0.3646	0.6319	1.0666	1.6199	2.1193	27.4976
0.08	0.0000	0.0164	0.0328	0.0812	0.1583	0.2297	0.3579	0.6259	1.0619	1.6165	2.1167	27.4976
0.09	0.0000	0.0158	0.0316	0.0785	0.1537	0.2241	0.3516	0.6200	1.0572	1.6130	2.1141	27.4976
0.10	0.0000	0.0153	0.0306	0.0761	0.1495	0.2189	0.3456	0.6142	1.0526	1.6096	2.1115	27.4976
						vertical displacement, *y*						
0.00	−0.0921	−0.0641	−0.0478	−0.0109	0.0360	0.0749	0.1403	0.2684	0.4567	0.6586	0.8062	1.3073
0.01	−0.0156	−0.0126	−0.0056	0.0202	0.0607	0.0965	0.1587	0.2834	0.4691	0.6695	0.8163	1.3157
0.02	0.0295	0.0308	0.0343	0.0518	0.0858	0.1185	0.1773	0.2985	0.4816	0.6804	0.8264	1.3241
0.03	0.0675	0.0683	0.0704	0.0828	0.1112	0.1406	0.1961	0.3136	0.4941	0.6913	0.8366	1.3325
0.04	0.1016	0.1021	0.1036	0.1128	0.1365	0.1629	0.2150	0.3288	0.5066	0.7023	0.8468	1.3409
0.05	0.1329	0.1333	0.1344	0.1416	0.1615	0.1853	0.2340	0.3442	0.5192	0.7133	0.8570	1.3493
0.06	0.1623	0.1626	0.1635	0.1693	0.1862	0.2075	0.2530	0.3595	0.5318	0.7243	0.8672	1.3577
0.07	0.1902	0.1905	0.1912	0.1960	0.2106	0.2297	0.2722	0.3750	0.5445	0.7353	0.8774	1.3661
0.08	0.2169	0.2171	0.2177	0.2217	0.2344	0.2517	0.2913	0.3905	0.5572	0.7464	0.8877	1.3745
0.09	0.2426	0.2427	0.2432	0.2467	0.2578	0.2734	0.3104	0.4060	0.5699	0.7574	0.8979	1.3829
0.10	0.2673	0.2675	0.2679	0.2709	0.2808	0.2949	0.3295	0.4216	0.5827	0.7685	0.9082	1.3913
						horizontal velocity, *u*						
0.00	0.0000	0.1460	0.1840	0.2501	0.3158	0.3621	0.4305	0.5451	0.6894	0.8277	0.9216	1.1908
0.01	0.1956	0.2032	0.2191	0.2665	0.3248	0.3684	0.4343	0.5467	0.6896	0.8272	0.9209	1.1908
0.02	0.2455	0.2481	0.2551	0.2860	0.3352	0.3754	0.4385	0.5484	0.6899	0.8268	0.9202	1.1908
0.03	0.2801	0.2814	0.2853	0.3060	0.3466	0.3831	0.4430	0.5502	0.6902	0.8264	0.9196	1.1908
0.04	0.3073	0.3081	0.3106	0.3252	0.3585	0.3913	0.4477	0.5522	0.6905	0.8260	0.9189	1.1908
0.05	0.3300	0.3306	0.3323	0.3431	0.3705	0.3998	0.4528	0.5542	0.6909	0.8256	0.9183	1.1908
0.06	0.3497	0.3501	0.3514	0.3597	0.3824	0.4085	0.4580	0.5564	0.6913	0.8252	0.9177	1.1908
0.07	0.3670	0.3674	0.3684	0.3750	0.3941	0.4172	0.4634	0.5586	0.6917	0.8249	0.9171	1.1908
0.08	0.3827	0.3830	0.3838	0.3892	0.4054	0.4260	0.4689	0.5609	0.6922	0.8245	0.9164	1.1908
0.09	0.3970	0.3972	0.3979	0.4024	0.4163	0.4347	0.4745	0.5633	0.6928	0.8242	0.9159	1.1908
0.10	0.4101	0.4103	0.4108	0.4146	0.4267	0.4432	0.4802	0.5658	0.6933	0.8239	0.9153	1.1908
						vertical velocity, *v*						
0.00	0.0000	0.0836	0.1048	0.1404	0.1734	0.1950	0.2237	0.2611	0.2842	0.2749	0.2466	0.0000
0.01	0.0000	0.0379	0.0665	0.1157	0.1571	0.1824	0.2146	0.2552	0.2805	0.2725	0.2449	0.0000
0.02	0.0000	0.0247	0.0471	0.0960	0.1422	0.1703	0.2056	0.2494	0.2769	0.2702	0.2433	0.0000
0.03	0.0000	0.0188	0.0368	0.0812	0.1289	0.1591	0.1970	0.2437	0.2733	0.2678	0.2416	0.0000
0.04	0.0000	0.0154	0.0304	0.0701	0.1173	0.1487	0.1888	0.2381	0.2697	0.2655	0.2399	0.0000
0.05	0.0000	0.0132	0.0261	0.0616	0.1072	0.1392	0.1808	0.2326	0.2661	0.2632	0.2383	0.0000
0.06	0.0000	0.0115	0.0229	0.0550	0.0985	0.1305	0.1733	0.2272	0.2626	0.2608	0.2366	0.0000
0.07	0.0000	0.0103	0.0205	0.0498	0.0910	0.1226	0.1661	0.2219	0.2591	0.2585	0.2349	0.0000
0.08	0.0000	0.0094	0.0186	0.0455	0.0844	0.1154	0.1593	0.2167	0.2556	0.2562	0.2333	0.0000
0.09	0.0000	0.0086	0.0171	0.0419	0.0787	0.1089	0.1528	0.2116	0.2522	0.2539	0.2316	0.0000
0.10	0.0000	0.0079	0.0158	0.0388	0.0737	0.1030	0.1467	0.2067	0.2488	0.2516	0.2300	0.0000

TABLE 8*d*. TIME, PRESSURE AND ACCELERATION NEAR THE SURFACE FOR $d = 0.2$

(ALSO APPLICABLE TO THE SOLITARY WAVE)

ψ/λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
						time, t						
0.00	0.0000	0.6653	0.8386	1.1395	1.4382	1.6491	1.9613	2.4872	3.1712	3.8864	4.4495	26.0860
0.01	0.0000	0.1580	0.2911	0.5710	0.8663	1.0769	1.3894	1.9171	2.6034	3.3207	3.8853	25.5263
0.02	0.0000	0.1031	0.2006	0.4421	0.7257	0.9340	1.2458	1.7747	2.4631	3.1824	3.7484	25.3938
0.03	0.0000	0.0797	0.1573	0.3656	0.6332	0.8372	1.1471	1.6763	2.3666	3.0879	3.6553	25.3051
0.04	0.0000	0.0664	0.1317	0.3143	0.5649	0.7634	1.0702	1.5991	2.2910	3.0142	3.5829	25.2372
0.05	0.0000	0.0576	0.1146	0.2776	0.5118	0.7040	1.0068	1.5347	2.2279	2.9530	3.5231	25.1818
0.06	0.0000	0.0513	0.1023	0.2500	0.4693	0.6548	0.9528	1.4790	2.1734	2.9003	3.4717	25.1348
0.07	0.0000	0.0466	0.0929	0.2284	0.4345	0.6131	0.9058	1.4299	2.1252	2.8538	3.4265	25.0939
0.08	0.0000	0.0429	0.0856	0.2111	0.4054	0.5774	0.8644	1.3857	2.0818	2.8120	3.3860	25.0577
0.09	0.0000	0.0399	0.0796	0.1968	0.3807	0.5464	0.8274	1.3457	2.0422	2.7740	3.3492	25.0253
0.10	0.0000	0.0373	0.0746	0.1849	0.3596	0.5192	0.7942	1.3089	2.0057	2.7391	3.3155	24.9959
						pressure, p						
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.01	0.0196	0.0189	0.0176	0.0147	0.0123	0.0111	0.0097	0.0082	0.0072	0.0065	0.0062	0.0043
0.02	0.0315	0.0312	0.0304	0.0274	0.0238	0.0217	0.0192	0.0164	0.0143	0.0131	0.0123	0.0085
0.03	0.0416	0.0415	0.0410	0.0385	0.0346	0.0319	0.0284	0.0245	0.0215	0.0196	0.0185	0.0128
0.04	0.0509	0.0508	0.0504	0.0485	0.0447	0.0416	0.0375	0.0325	0.0285	0.0261	0.0247	0.0170
0.05	0.0595	0.0594	0.0592	0.0576	0.0541	0.0509	0.0463	0.0404	0.0356	0.0326	0.0308	0.0213
0.06	0.0678	0.0677	0.0675	0.0662	0.0630	0.0599	0.0550	0.0482	0.0427	0.0391	0.0370	0.0255
0.07	0.0757	0.0756	0.0754	0.0744	0.0715	0.0685	0.0634	0.0560	0.0497	0.0456	0.0431	0.0298
0.08	0.0833	0.0833	0.0831	0.0822	0.0797	0.0768	0.0716	0.0637	0.0567	0.0520	0.0492	0.0340
0.09	0.0907	0.0907	0.0906	0.0898	0.0875	0.0848	0.0797	0.0713	0.0636	0.0585	0.0554	0.0383
0.10	0.0980	0.0980	0.0979	0.0972	0.0952	0.0926	0.0875	0.0788	0.0705	0.0649	0.0615	0.0425
						horizontal acceleration						
0.00	0.0000	0.2196	0.2197	0.2199	0.2199	0.2198	0.2191	0.2159	0.2044	0.1801	0.1524	0.0000
0.01	0.0000	0.0920	0.1420	0.1881	0.2041	0.2091	0.2125	0.2122	0.2024	0.1790	0.1518	0.0000
0.02	0.0000	0.0505	0.0915	0.1555	0.1871	0.1978	0.2056	0.2085	0.2004	0.1779	0.1512	0.0000
0.03	0.0000	0.0342	0.0651	0.1280	0.1702	0.1862	0.1985	0.2048	0.1984	0.1768	0.1507	0.0000
0.04	0.0000	0.0257	0.0500	0.1065	0.1540	0.1746	0.1913	0.2010	0.1963	0.1757	0.1501	0.0000
0.05	0.0000	0.0205	0.0403	0.0902	0.1393	0.1633	0.1841	0.1972	0.1943	0.1746	0.1495	0.0000
0.06	0.0000	0.0171	0.0337	0.0777	0.1262	0.1526	0.1769	0.1933	0.1922	0.1735	0.1489	0.0000
0.07	0.0000	0.0146	0.0289	0.0679	0.1147	0.1425	0.1698	0.1895	0.1902	0.1724	0.1483	0.0000
0.08	0.0000	0.0127	0.0252	0.0601	0.1046	0.1331	0.1628	0.1856	0.1881	0.1713	0.1477	0.0000
0.09	0.0000	0.0113	0.0224	0.0538	0.0959	0.1245	0.1561	0.1817	0.1861	0.1702	0.1471	0.0000
0.10	0.0000	0.0101	0.0201	0.0487	0.0882	0.1166	0.1496	0.1779	0.1840	0.1691	0.1466	0.0000
						vertical acceleration						
0.00	0.2531	0.1234	0.1210	0.1149	0.1063	0.0986	0.0849	0.0560	0.0108	−0.0356	−0.0629	0.0000
0.01	0.2464	0.2284	0.2005	0.1552	0.1274	0.1128	0.0934	0.0602	0.0131	−0.0341	−0.0617	0.0000
0.02	0.2413	0.2359	0.2230	0.1824	0.1455	0.1256	0.1014	0.0643	0.0154	−0.0326	−0.0605	0.0000
0.03	0.2369	0.2343	0.2275	0.1980	0.1599	0.1369	0.1088	0.0683	0.0175	−0.0312	−0.0593	0.0000
0.04	0.2327	0.2313	0.2272	0.2060	0.1709	0.1465	0.1157	0.0721	0.0197	−0.0297	−0.0582	0.0000
0.05	0.2289	0.2279	0.2252	0.2096	0.1788	0.1544	0.1219	0.0757	0.0217	−0.0283	−0.0570	0.0000
0.06	0.2252	0.2245	0.2226	0.2108	0.1843	0.1608	0.1274	0.0791	0.0238	−0.0269	−0.0559	0.0000
0.07	0.2217	0.2212	0.2198	0.2106	0.1880	0.1659	0.1323	0.0824	0.0257	−0.0255	−0.0548	0.0000
0.08	0.2184	0.2180	0.2169	0.2096	0.1902	0.1698	0.1366	0.0855	0.0276	−0.0241	−0.0537	0.0000
0.09	0.2152	0.2149	0.2140	0.2080	0.1914	0.1726	0.1403	0.0884	0.0295	−0.0228	−0.0526	0.0000
0.10	0.2121	0.2118	0.2111	0.2061	0.1917	0.1746	0.1435	0.0911	0.0313	−0.0215	−0.0515	0.0000

TABLE 8*e*. MEAN DEPTH AS A FUNCTION OF ψ FOR $d = 0.2$

$\psi = 0(0.01)0.1$	1.7806	1.7712	1.7620	1.7528	1.7436	1.7344	1.7253	1.7162	1.7070	1.6979	1.6888
$\psi = 0(0.2)2.0$	1.7806	1.5983	1.4188	1.2403	1.0624	0.8849	0.7076	0.5306	0.3537	0.1768	0.0000

Table 9a. Displacement and velocity for $d = 0.5$

($1/2F^2 = 0.540988$, $h = 1.87625$, $L = 22.8320$, $\bar{y}_s = 1.04798$.)

/λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
ψ					horizontal displacement, x						
0.0	0.0000	1.7739	2.9851	4.1014	5.1763	6.2305	7.2742	8.3125	9.3481	10.3823	11.4160
0.2	0.0000	1.7135	2.9548	4.0859	5.1683	6.2264	7.2721	8.3115	9.3476	10.3821	11.4160
0.4	0.0000	1.6609	2.9271	4.0715	5.1608	6.2225	7.2701	8.3104	9.3471	10.3819	11.4160
0.6	0.0000	1.6163	2.9022	4.0584	5.1541	6.2191	7.2683	8.3095	9.3466	10.3817	11.4160
0.8	0.0000	1.5796	2.8803	4.0468	5.1480	6.2159	7.2667	8.3087	9.3462	10.3816	11.4160
1.0	0.0000	1.5501	2.8616	4.0368	5.1428	6.2133	7.2653	8.3080	9.3459	10.3814	11.4160
1.2	0.0000	1.5272	2.8463	4.0285	5.1385	6.2110	7.2642	8.3074	9.3456	10.3813	11.4160
1.4	0.0000	1.5100	2.8343	4.0220	5.1351	6.2092	7.2633	8.3070	9.3454	10.3812	11.4160
1.6	0.0000	1.4982	2.8257	4.0173	5.1326	6.2080	7.2626	8.3066	9.3452	10.3812	11.4160
1.8	0.0000	1.4913	2.8206	4.0144	5.1311	6.2072	7.2622	8.3064	9.3451	10.3811	11.4160
2.0	0.0000	1.4890	2.8188	4.0134	5.1306	6.2069	7.2621	8.3064	9.3451	10.3811	11.4160
					vertical displacement, y						
0.0	−0.0881	0.6991	0.9846	1.1283	1.2022	1.2403	1.2599	1.2700	1.2750	1.2774	1.2780
0.2	0.4762	0.9098	1.1692	1.3028	1.3717	1.4073	1.4256	1.4350	1.4397	1.4419	1.4425
0.4	0.8250	1.1266	1.3568	1.4787	1.5420	1.5747	1.5916	1.6002	1.6045	1.6065	1.6071
0.6	1.1291	1.3477	1.5472	1.6562	1.7131	1.7425	1.7577	1.7654	1.7693	1.7711	1.7717
0.8	1.4104	1.5713	1.7399	1.8349	1.8848	1.9106	1.9240	1.9308	1.9342	1.9358	1.9363
1.0	1.6776	1.7963	1.9345	2.0147	2.0571	2.0791	2.0904	2.0962	2.0992	2.1005	2.1009
1.2	1.9357	2.0219	2.1306	2.1955	2.2299	2.2478	2.2570	2.2618	2.2641	2.2652	2.2655
1.4	2.1875	2.2477	2.3280	2.3770	2.4031	2.4167	2.4237	2.4273	2.4291	2.4300	2.4302
1.6	2.4352	2.4733	2.5263	2.5591	2.5767	2.5858	2.5905	2.5929	2.5941	2.5947	2.5949
1.8	2.6803	2.6988	2.7251	2.7416	2.7504	2.7550	2.7574	2.7586	2.7592	2.7595	2.7596
2.0	2.9242	2.9242	2.9242	2.9242	2.9242	2.9242	2.9242	2.9242	2.9242	2.9242	2.9242
					horizontal velocity, u						
0.0	0.0000	0.8818	1.0618	1.1424	1.1802	1.1985	1.2076	1.2122	1.2144	1.2155	1.2158
0.2	0.5157	0.8731	1.0486	1.1331	1.1747	1.1955	1.2060	1.2113	1.2140	1.2152	1.2156
0.4	0.6235	0.8696	1.0376	1.1248	1.1697	1.1927	1.2045	1.2105	1.2135	1.2149	1.2153
0.6	0.6881	0.8697	1.0286	1.1176	1.1652	1.1902	1.2031	1.2098	1.2131	1.2147	1.2151
0.8	0.7319	0.8721	1.0213	1.1114	1.1613	1.1880	1.2019	1.2092	1.2128	1.2145	1.2150
1.0	0.7632	0.8755	1.0156	1.1062	1.1580	1.1861	1.2009	1.2086	1.2125	1.2143	1.2148
1.2	0.7857	0.8792	1.0112	1.1020	1.1553	1.1845	1.2000	1.2081	1.2122	1.2141	1.2147
1.4	0.8017	0.8825	1.0080	1.0988	1.1531	1.1833	1.1994	1.2078	1.2120	1.2140	1.2146
1.6	0.8125	0.8851	1.0059	1.0965	1.1516	1.1824	1.1989	1.2075	1.2119	1.2139	1.2145
1.8	0.8186	0.8867	1.0046	1.0952	1.1507	1.1819	1.1986	1.2074	1.2118	1.2139	1.2145
2.0	0.8207	0.8873	1.0042	1.0947	1.1504	1.1817	1.1985	1.2073	1.2118	1.2138	1.2144
					vertical velocity, v						
0.0	0.0000	0.2726	0.1823	0.1059	0.0579	0.0307	0.0160	0.0082	0.0040	0.0016	0.0000
0.2	0.0000	0.2310	0.1636	0.0970	0.0535	0.0285	0.0149	0.0077	0.0038	0.0015	0.0000
0.4	0.0000	0.1928	0.1447	0.0875	0.0487	0.0261	0.0137	0.0070	0.0035	0.0014	0.0000
0.6	0.0000	0.1585	0.1258	0.0775	0.0435	0.0234	0.0123	0.0063	0.0031	0.0013	0.0000
0.8	0.0000	0.1282	0.1070	0.0670	0.0379	0.0205	0.0108	0.0056	0.0027	0.0011	0.0000
1.0	0.0000	0.1014	0.0885	0.0563	0.0321	0.0174	0.0092	0.0047	0.0023	0.0009	0.0000
1.2	0.0000	0.0776	0.0703	0.0453	0.0259	0.0141	0.0074	0.0038	0.0019	0.0008	0.0000
1.4	0.0000	0.0562	0.0524	0.0341	0.0196	0.0107	0.0056	0.0029	0.0014	0.0006	0.0000
1.6	0.0000	0.0365	0.0348	0.0228	0.0132	0.0072	0.0038	0.0020	0.0010	0.0004	0.0000
1.8	0.0000	0.0180	0.0173	0.0114	0.0066	0.0036	0.0019	0.0010	0.0005	0.0002	0.0000
2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

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Table 9*b*. Time, pressure and acceleration for $d = 0.5$

/λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
ψ						time, t					
0.0	0.0000	3.9107	5.1470	6.1561	7.0799	7.9656	8.8328	9.6908	10.5442	11.3954	12.2456
−0.2	0.0000	2.5796	3.8631	4.8962	5.8325	6.7245	7.5950	8.4547	9.3090	10.1607	11.0114
−0.4	0.0000	2.3072	3.6288	4.6837	5.6314	6.5293	7.4028	8.2642	9.1193	9.9715	10.8225
−0.6	0.0000	2.1368	3.4875	4.5617	5.5197	6.4230	7.2993	8.1621	9.0180	9.8706	10.7219
−0.8	0.0000	2.0170	3.3888	4.4797	5.4469	6.3550	7.2338	8.0978	8.9544	9.8074	10.6590
−1.0	0.0000	1.9296	3.3161	4.4211	5.3961	6.3084	7.1893	8.0545	8.9117	9.7650	10.6168
−1.2	0.0000	1.8655	3.2619	4.3783	5.3599	6.2756	7.1583	8.0244	8.8821	9.7357	10.5877
−1.4	0.0000	1.8198	3.2224	4.3476	5.3343	6.2527	7.1369	8.0037	8.8618	9.7156	10.5678
−1.6	0.0000	1.7890	3.1954	4.3268	5.3173	6.2376	7.1228	7.9902	8.8485	9.7025	10.5547
−1.8	0.0000	1.7712	3.1797	4.3148	5.3074	6.2290	7.1148	7.9824	8.8410	9.6951	10.5473
−2.0	0.0000	1.7654	3.1745	4.3108	5.3042	6.2261	7.1121	7.9799	8.8385	9.6927	10.5450
						pressure, p					
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−0.2	0.1723	0.1321	0.1170	0.1058	0.0984	0.0940	0.0916	0.0903	0.0897	0.0894	0.0893
−0.4	0.2996	0.2605	0.2329	0.2112	0.1966	0.1880	0.1832	0.1807	0.1794	0.1787	0.1786
−0.6	0.4218	0.3860	0.3478	0.3161	0.2946	0.2818	0.2747	0.2709	0.2690	0.2681	0.2678
−0.8	0.5428	0.5093	0.4617	0.4205	0.3923	0.3755	0.3661	0.3612	0.3586	0.3575	0.3571
−1.0	0.6640	0.6310	0.5746	0.5241	0.4895	0.4689	0.4574	0.4514	0.4482	0.4468	0.4464
−1.2	0.7862	0.7520	0.6866	0.6271	0.5864	0.5621	0.5486	0.5415	0.5378	0.5361	0.5356
−1.4	0.9097	0.8726	0.7977	0.7293	0.6827	0.6550	0.6396	0.6315	0.6273	0.6254	0.6248
−1.6	1.0350	0.9933	0.9079	0.8307	0.7784	0.7475	0.7305	0.7214	0.7167	0.7146	0.7140
−1.8	1.1626	1.1144	1.0172	0.9311	0.8735	0.8397	0.8211	0.8112	0.8061	0.8038	0.8031
−2.0	1.2929	1.2360	1.1255	1.0305	0.9680	0.9315	0.9115	0.9009	0.8955	0.8929	0.8922
						horizontal acceleration					
0.0	0.0000	0.1826	0.1072	0.0561	0.0285	0.0144	0.0073	0.0037	0.0018	0.0007	0.0000
−0.2	0.0000	0.1632	0.1061	0.0596	0.0317	0.0165	0.0085	0.0043	0.0021	0.0009	0.0000
−0.4	0.0000	0.1440	0.1041	0.0623	0.0344	0.0183	0.0096	0.0049	0.0024	0.0010	0.0000
−0.6	0.0000	0.1262	0.1014	0.0643	0.0367	0.0199	0.0105	0.0054	0.0027	0.0011	0.0000
−0.8	0.0000	0.1106	0.0984	0.0658	0.0387	0.0213	0.0114	0.0059	0.0029	0.0012	0.0000
−1.0	0.0000	0.0978	0.0954	0.0668	0.0403	0.0225	0.0121	0.0063	0.0031	0.0013	0.0000
−1.2	0.0000	0.0875	0.0927	0.0675	0.0416	0.0235	0.0127	0.0066	0.0033	0.0013	0.0000
−1.4	0.0000	0.0799	0.0904	0.0680	0.0426	0.0242	0.0131	0.0069	0.0034	0.0014	0.0000
−1.6	0.0000	0.0746	0.0887	0.0683	0.0432	0.0248	0.0135	0.0070	0.0035	0.0014	0.0000
−1.8	0.0000	0.0715	0.0876	0.0684	0.0436	0.0251	0.0137	0.0072	0.0035	0.0015	0.0000
−2.0	0.0000	0.0705	0.0873	0.0685	0.0438	0.0252	0.0137	0.0072	0.0036	0.0015	0.0000
						vertical acceleration					
0.0	0.2697	−0.0498	−0.0831	−0.0645	−0.0400	−0.0226	−0.0122	−0.0065	−0.0036	−0.0022	−0.0018
−0.2	0.1979	−0.0239	−0.0680	−0.0567	−0.0363	−0.0208	−0.0113	−0.0061	−0.0034	−0.0021	−0.0017
−0.4	0.1555	−0.0057	−0.0548	−0.0492	−0.0324	−0.0188	−0.0103	−0.0056	−0.0031	−0.0019	−0.0015
−0.6	0.1237	0.0058	−0.0435	−0.0420	−0.0284	−0.0167	−0.0093	−0.0050	−0.0028	−0.0017	−0.0014
−0.8	0.0982	0.0120	−0.0340	−0.0352	−0.0244	−0.0145	−0.0081	−0.0044	−0.0024	−0.0015	−0.0012
−1.0	0.0770	0.0143	−0.0260	−0.0287	−0.0203	−0.0122	−0.0068	−0.0037	−0.0021	−0.0013	−0.0010
−1.2	0.0587	0.0139	−0.0193	−0.0226	−0.0163	−0.0099	−0.0055	−0.0030	−0.0017	−0.0010	−0.0008
−1.4	0.0425	0.0117	−0.0136	−0.0167	−0.0122	−0.0074	−0.0042	−0.0023	−0.0013	−0.0008	−0.0006
−1.6	0.0276	0.0084	−0.0086	−0.0110	−0.0081	−0.0050	−0.0028	−0.0015	−0.0009	−0.0005	−0.0004
−1.8	0.0136	0.0043	−0.0042	−0.0055	−0.0041	−0.0025	−0.0014	−0.0008	−0.0004	−0.0003	−0.0002
−2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE 9c. DISPLACEMENT AND VELOCITY NEAR THE SURFACE FOR $d = 0.5$

φ/λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ							horizontal displacement, x					
0.00	0.0000	0.0258	0.0409	0.0755	0.1200	0.1575	0.2220	0.3547	0.5692	0.8385	1.0773	11.4160
-0.01	0.0000	0.0125	0.0247	0.0581	0.1042	0.1431	0.2096	0.3448	0.5616	0.8326	1.0723	11.4160
-0.02	0.0000	0.0100	0.0199	0.0488	0.0926	0.1313	0.1984	0.3354	0.5542	0.8267	1.0674	11.4160
-0.03	0.0000	0.0088	0.0175	0.0434	0.0842	0.1218	0.1886	0.3265	0.5470	0.8209	1.0625	11.4160
-0.04	0.0000	0.0080	0.0160	0.0397	0.0780	0.1142	0.1799	0.3181	0.5400	0.8152	1.0577	11.4160
-0.05	0.0000	0.0074	0.0149	0.0371	0.0733	0.1080	0.1724	0.3103	0.5331	0.8096	1.0529	11.4160
-0.06	0.0000	0.0070	0.0141	0.0351	0.0695	0.1029	0.1658	0.3029	0.5265	0.8041	1.0482	11.4160
-0.07	0.0000	0.0067	0.0134	0.0334	0.0664	0.0986	0.1600	0.2960	0.5200	0.7986	1.0435	11.4160
-0.08	0.0000	0.0064	0.0128	0.0321	0.0638	0.0950	0.1549	0.2896	0.5138	0.7933	1.0389	11.4160
-0.09	0.0000	0.0062	0.0124	0.0309	0.0616	0.0918	0.1503	0.2836	0.5078	0.7880	1.0344	11.4160
-0.10	0.0000	0.0060	0.0120	0.0299	0.0597	0.0891	0.1463	0.2780	0.5019	0.7829	1.0298	11.4160
							vertical displacement, y					
0.00	−0.0881	−0.0733	−0.0646	−0.0448	−0.0196	0.0015	0.0372	0.1087	0.2185	0.3457	0.4487	1.2780
-0.01	−0.0133	−0.0128	−0.0113	−0.0035	0.0133	0.0301	0.0613	0.1280	0.2342	0.3591	0.4609	1.2863
-0.02	0.0309	0.0311	0.0317	0.0355	0.0463	0.0592	0.0859	0.1476	0.2499	0.3725	0.4731	1.2945
-0.03	0.0680	0.0681	0.0685	0.0709	0.0783	0.0882	0.1107	0.1673	0.2657	0.3859	0.4853	1.3027
-0.04	0.1013	0.1014	0.1016	0.1033	0.1087	0.1164	0.1355	0.1872	0.2817	0.3995	0.4976	1.3109
-0.05	0.1320	0.1321	0.1322	0.1335	0.1376	0.1438	0.1600	0.2072	0.2977	0.4130	0.5099	1.3192
-0.06	0.1608	0.1608	0.1610	0.1619	0.1652	0.1703	0.1842	0.2272	0.3138	0.4267	0.5222	1.3274
-0.07	0.1881	0.1881	0.1882	0.1890	0.1917	0.1960	0.2079	0.2472	0.3300	0.4403	0.5346	1.3356
-0.08	0.2142	0.2142	0.2143	0.2149	0.2172	0.2209	0.2313	0.2671	0.3462	0.4541	0.5471	1.3438
-0.09	0.2393	0.2393	0.2394	0.2399	0.2419	0.2450	0.2542	0.2870	0.3625	0.4678	0.5595	1.3521
-0.10	0.2635	0.2635	0.2636	0.2641	0.2658	0.2685	0.2767	0.3068	0.3788	0.4816	0.5720	1.3603
							horizontal velocity, u					
0.00	0.0000	0.1098	0.1384	0.1881	0.2372	0.2718	0.3228	0.4080	0.5164	0.6247	0.7046	1.2158
-0.01	0.1999	0.2013	0.2051	0.2239	0.2575	0.2862	0.3320	0.4129	0.5186	0.6256	0.7048	1.2158
-0.02	0.2509	0.2514	0.2527	0.2608	0.2813	0.3031	0.3427	0.4182	0.5210	0.6265	0.7051	1.2158
-0.03	0.2863	0.2865	0.2872	0.2916	0.3048	0.3211	0.3543	0.4241	0.5236	0.6275	0.7054	1.2158
-0.04	0.3141	0.3142	0.3146	0.3175	0.3265	0.3388	0.3664	0.4303	0.5263	0.6286	0.7058	1.2157
-0.05	0.3373	0.3374	0.3377	0.3397	0.3463	0.3557	0.3787	0.4368	0.5292	0.6297	0.7061	1.2157
-0.06	0.3574	0.3575	0.3577	0.3592	0.3642	0.3717	0.3909	0.4436	0.5322	0.6309	0.7066	1.2157
-0.07	0.3752	0.3752	0.3754	0.3765	0.3805	0.3866	0.4028	0.4505	0.5354	0.6322	0.7070	1.2157
-0.08	0.3912	0.3912	0.3913	0.3923	0.3955	0.4005	0.4144	0.4575	0.5386	0.6335	0.7075	1.2157
-0.09	0.4057	0.4058	0.4059	0.4067	0.4093	0.4135	0.4255	0.4646	0.5420	0.6349	0.7081	1.2157
-0.10	0.4192	0.4192	0.4193	0.4199	0.4222	0.4258	0.4362	0.4717	0.5454	0.6363	0.7086	1.2157
							vertical velocity, v					
0.00	0.0000	0.0632	0.0795	0.1072	0.1338	0.1518	0.1773	0.2158	0.2553	0.2814	0.2905	0.0000
-0.01	0.0000	0.0164	0.0317	0.0680	0.1053	0.1291	0.1606	0.2049	0.2483	0.2766	0.2867	0.0000
-0.02	0.0000	0.0103	0.0203	0.0481	0.0842	0.1101	0.1453	0.1945	0.2415	0.2719	0.2830	0.0000
-0.03	0.0000	0.0077	0.0154	0.0376	0.0695	0.0950	0.1318	0.1846	0.2348	0.2672	0.2793	0.0000
-0.04	0.0000	0.0063	0.0126	0.0311	0.0592	0.0831	0.1199	0.1752	0.2283	0.2625	0.2757	0.0000
-0.05	0.0000	0.0054	0.0108	0.0267	0.0516	0.0737	0.1096	0.1663	0.2219	0.2579	0.2720	0.0000
-0.06	0.0000	0.0047	0.0095	0.0235	0.0458	0.0662	0.1007	0.1579	0.2157	0.2534	0.2684	0.0000
-0.07	0.0000	0.0042	0.0084	0.0210	0.0412	0.0601	0.0930	0.1501	0.2096	0.2489	0.2648	0.0000
-0.08	0.0000	0.0038	0.0077	0.0191	0.0376	0.0551	0.0863	0.1429	0.2037	0.2445	0.2613	0.0000
-0.09	0.0000	0.0035	0.0070	0.0175	0.0345	0.0508	0.0804	0.1361	0.1980	0.2401	0.2578	0.0000
-0.10	0.0000	0.0032	0.0065	0.0161	0.0320	0.0472	0.0753	0.1298	0.1924	0.2359	0.2543	0.0000

Table 9d. Time, pressure and acceleration near the surface for d = 0.5

/λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ						time, t						
0.00	0.0000	0.4695	0.5915	0.8031	1.0126	1.1599	1.3772	1.7403	2.2044	2.6762	3.0354	12.2456
−0.01	0.0000	0.0625	0.1226	0.2787	0.4707	0.6139	0.8292	1.1925	1.6582	2.1317	2.4920	11.7100
−0.02	0.0000	0.0398	0.0793	0.1920	0.3538	0.4864	0.6946	1.0552	1.5213	1.9962	2.3576	11.5831
−0.03	0.0000	0.0306	0.0611	0.1506	0.2878	0.4079	0.6060	0.9612	1.4269	1.9028	2.2653	11.4983
−0.04	0.0000	0.0255	0.0509	0.1261	0.2451	0.3538	0.5407	0.8884	1.3527	1.8295	2.1929	11.4332
−0.05	0.0000	0.0221	0.0441	0.1097	0.2153	0.3142	0.4899	0.8290	1.2910	1.7683	2.1325	11.3802
−0.06	0.0000	0.0197	0.0393	0.0979	0.1932	0.2840	0.4492	0.7788	1.2379	1.7154	2.0804	11.3352
−0.07	0.0000	0.0179	0.0357	0.0890	0.1762	0.2602	0.4159	0.7356	1.1910	1.6685	2.0342	11.2961
−0.08	0.0000	0.0164	0.0328	0.0819	0.1625	0.2409	0.3880	0.6979	1.1491	1.6264	1.9926	11.2615
−0.09	0.0000	0.0153	0.0305	0.0762	0.1514	0.2249	0.3644	0.6647	1.1112	1.5879	1.9547	11.2304
−0.10	0.0000	0.0143	0.0286	0.0714	0.1421	0.2114	0.3442	0.6351	1.0767	1.5526	1.9199	11.2023
						pressure, p						
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−0.01	0.0205	0.0204	0.0200	0.0184	0.0162	0.0147	0.0129	0.0107	0.0090	0.0080	0.0075	0.0045
−0.02	0.0329	0.0328	0.0327	0.0317	0.0296	0.0277	0.0249	0.0211	0.0180	0.0160	0.0150	0.0089
−0.03	0.0435	0.0435	0.0434	0.0428	0.0412	0.0393	0.0361	0.0312	0.0268	0.0239	0.0224	0.0134
−0.04	0.0532	0.0531	0.0531	0.0527	0.0514	0.0498	0.0466	0.0410	0.0355	0.0318	0.0298	0.0179
−0.05	0.0622	0.0622	0.0621	0.0618	0.0608	0.0595	0.0565	0.0505	0.0441	0.0396	0.0372	0.0223
−0.06	0.0708	0.0708	0.0707	0.0705	0.0697	0.0686	0.0658	0.0597	0.0526	0.0474	0.0446	0.0268
−0.07	0.0790	0.0790	0.0790	0.0788	0.0782	0.0772	0.0747	0.0687	0.0609	0.0551	0.0519	0.0313
−0.08	0.0870	0.0870	0.0870	0.0868	0.0863	0.0854	0.0832	0.0773	0.0692	0.0628	0.0592	0.0357
−0.09	0.0948	0.0948	0.0948	0.0946	0.0942	0.0934	0.0914	0.0858	0.0773	0.0704	0.0665	0.0402
−0.10	0.1024	0.1024	0.1024	0.1022	0.1018	0.1012	0.0994	0.0940	0.0853	0.0780	0.0737	0.0446
						horizontal acceleration						
0.00	0.0000	0.2343	0.2345	0.2347	0.2348	0.2348	0.2348	0.2344	0.2322	0.2263	0.2182	0.0000
−0.01	0.0000	0.0439	0.0821	0.1517	0.1922	0.2067	0.2180	0.2257	0.2275	0.2234	0.2161	0.0000
−0.02	0.0000	0.0223	0.0437	0.0977	0.1509	0.1770	0.1999	0.2166	0.2226	0.2204	0.2139	0.0000
−0.03	0.0000	0.0148	0.0294	0.0695	0.1194	0.1502	0.1817	0.2072	0.2177	0.2174	0.2118	0.0000
−0.04	0.0000	0.0111	0.0220	0.0534	0.0968	0.1279	0.1645	0.1977	0.2128	0.2144	0.2096	0.0000
−0.05	0.0000	0.0088	0.0176	0.0431	0.0806	0.1101	0.1488	0.1883	0.2078	0.2114	0.2074	0.0000
−0.06	0.0000	0.0073	0.0146	0.0360	0.0686	0.0959	0.1348	0.1790	0.2028	0.2084	0.2053	0.0000
−0.07	0.0000	0.0062	0.0125	0.0309	0.0595	0.0845	0.1225	0.1700	0.1977	0.2053	0.2031	0.0000
−0.08	0.0000	0.0054	0.0109	0.0270	0.0524	0.0753	0.1117	0.1613	0.1927	0.2023	0.2009	0.0000
−0.09	0.0000	0.0048	0.0096	0.0239	0.0468	0.0677	0.1024	0.1531	0.1877	0.1992	0.1987	0.0000
−0.10	0.0000	0.0043	0.0086	0.0215	0.0421	0.0614	0.0942	0.1453	0.1828	0.1962	0.1965	0.0000
						vertical acceleration						
0.00	0.2697	0.1336	0.1324	0.1292	0.1247	0.1207	0.1134	0.0978	0.0714	0.0387	0.0116	−0.0018
−0.01	0.2631	0.2594	0.2498	0.2142	0.1768	0.1573	0.1360	0.1091	0.0771	0.0420	0.0140	−0.0018
−0.02	0.2577	0.2567	0.2539	0.2381	0.2074	0.1842	0.1553	0.1197	0.0825	0.0452	0.0164	−0.0018
−0.03	0.2529	0.2525	0.2512	0.2430	0.2221	0.2015	0.1707	0.1293	0.0877	0.0484	0.0187	−0.0018
−0.04	0.2485	0.2483	0.2475	0.2426	0.2283	0.2116	0.1824	0.1378	0.0926	0.0514	0.0210	−0.0018
−0.05	0.2444	0.2442	0.2437	0.2405	0.2303	0.2171	0.1909	0.1454	0.0972	0.0543	0.0232	−0.0018
−0.06	0.2405	0.2404	0.2400	0.2377	0.2301	0.2197	0.1968	0.1519	0.1016	0.0571	0.0254	−0.0018
−0.07	0.2368	0.2367	0.2364	0.2347	0.2289	0.2205	0.2007	0.1575	0.1057	0.0599	0.0275	−0.0018
−0.08	0.2332	0.2331	0.2329	0.2316	0.2270	0.2201	0.2031	0.1621	0.1095	0.0625	0.0295	−0.0018
−0.09	0.2298	0.2297	0.2296	0.2285	0.2247	0.2191	0.2043	0.1659	0.1130	0.0650	0.0315	−0.0017
−0.10	0.2264	0.2264	0.2263	0.2254	0.2223	0.2175	0.2047	0.1689	0.1162	0.0674	0.0334	−0.0017

Table 9e. Mean depth as a function of ψ for d = 0.5

$-\psi$ = 0(0.01)0.1	1.8763	1.8657	1.8556	1.8455	1.8355	1.8255	1.8156	1.8057	1.7958	1.7860	1.7761
$-\psi$ = 0(0.2)2.0	1.8763	1.6789	1.4879	1.2992	1.1120	0.9256	0.7399	0.5546	0.3696	0.1848	0.0000

Table 10*a*. Displacement and velocity for $d = 1.0$

($1/2F^2 = 0.656518$, $h = 1.94110$, $L = 11.72972$, $\bar{y}_s = 0.82049$.)

/λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
ψ						horizontal displacement, x					
0.0	0.0000	1.0104	1.6641	2.2497	2.8015	3.3331	3.8516	4.3613	4.8653	5.3659	5.8649
0.2	0.0000	0.9265	1.6077	2.2103	2.7737	3.3135	3.8380	4.3523	4.8598	5.3633	5.8649
0.4	0.0000	0.8626	1.5586	2.1747	2.7482	3.2955	3.8255	4.3439	4.8547	5.3609	5.8649
0.6	0.0000	0.8159	1.5170	2.1432	2.7253	3.2792	3.8141	4.3363	4.8500	5.3587	5.8649
0.8	0.0000	0.7822	1.4828	2.1161	2.7052	3.2647	3.8040	4.3295	4.8459	5.3567	5.8649
1.0	0.0000	0.7577	1.4553	2.0933	2.6880	3.2523	3.7953	4.3237	4.8423	5.3550	5.8649
1.2	0.0000	0.7402	1.4339	2.0749	2.6739	3.2420	3.7881	4.3188	4.8394	5.3536	5.8649
1.4	0.0000	0.7278	1.4180	2.0608	2.6629	3.2339	3.7824	4.3150	4.8370	5.3525	5.8649
1.6	0.0000	0.7196	1.4070	2.0508	2.6550	3.2281	3.7783	4.3123	4.8353	5.3517	5.8649
1.8	0.0000	0.7149	1.4005	2.0448	2.6503	3.2246	3.7758	4.3106	4.8343	5.3512	5.8649
2.0	0.0000	0.7134	1.3984	2.0429	2.6487	3.2235	3.7750	4.3100	4.8339	5.3510	5.8649
						vertical displacement, y					
0.0	−0.0652	0.4378	0.6718	0.8280	0.9369	1.0134	1.0667	1.1031	1.1265	1.1396	1.1438
0.2	0.4639	0.6719	0.8696	1.0103	1.1104	1.1813	1.2310	1.2649	1.2868	1.2990	1.3029
0.4	0.7911	0.9133	1.0732	1.1966	1.2868	1.3514	1.3969	1.4280	1.4480	1.4593	1.4629
0.6	1.0764	1.1548	1.2807	1.3864	1.4658	1.5235	1.5642	1.5921	1.6102	1.6203	1.6235
0.8	1.3404	1.3933	1.4907	1.5788	1.6471	1.6972	1.7328	1.7573	1.7731	1.7820	1.7848
1.0	1.5912	1.6280	1.7020	1.7733	1.8301	1.8723	1.9025	1.9233	1.9368	1.9443	1.9467
1.2	1.8335	1.8591	1.9139	1.9694	2.0147	2.0487	2.0732	2.0901	2.1010	2.1071	2.1091
1.4	2.0699	2.0872	2.1260	2.1666	2.2005	2.2261	2.2446	2.2574	2.2657	2.2704	2.2719
1.6	2.3024	2.3132	2.3380	2.3646	2.3870	2.4042	2.4166	2.4252	2.4308	2.4339	2.4349
1.8	2.5326	2.5378	2.5498	2.5630	2.5742	2.5828	2.5890	2.5933	2.5961	2.5977	2.5982
2.0	2.7616	2.7616	2.7616	2.7616	2.7616	2.7616	2.7616	2.7616	2.7616	2.7616	2.7616
						horizontal velocity, u					
0.0	0.0000	0.7516	0.9403	1.0557	1.1312	1.1814	1.2147	1.2365	1.2501	1.2576	1.2599
0.2	0.5498	0.7636	0.9308	1.0412	1.1166	1.1685	1.2039	1.2275	1.2425	1.2508	1.2535
0.4	0.6647	0.7857	0.9267	1.0302	1.1044	1.1572	1.1942	1.2194	1.2356	1.2447	1.2476
0.6	0.7333	0.8103	0.9266	1.0220	1.0944	1.1476	1.1858	1.2122	1.2294	1.2391	1.2422
0.8	0.7799	0.8331	0.9289	1.0163	1.0862	1.1394	1.1785	1.2059	1.2240	1.2342	1.2375
1.0	0.8131	0.8524	0.9324	1.0124	1.0798	1.1327	1.1724	1.2006	1.2193	1.2300	1.2335
1.2	0.8370	0.8678	0.9361	1.0098	1.0749	1.1274	1.1674	1.1962	1.2155	1.2265	1.2301
1.4	0.8540	0.8795	0.9395	1.0083	1.0713	1.1233	1.1635	1.1928	1.2125	1.2238	1.2275
1.6	0.8653	0.8876	0.9422	1.0073	1.0688	1.1205	1.1608	1.1904	1.2104	1.2218	1.2256
1.8	0.8719	0.8924	0.9439	1.0069	1.0674	1.1188	1.1592	1.1889	1.2091	1.2207	1.2244
2.0	0.8740	0.8940	0.9444	1.0067	1.0669	1.1182	1.1587	1.1885	1.2086	1.2203	1.2241
						vertical velocity, v					
0.0	0.0000	0.3091	0.2890	0.2414	0.1901	0.1436	0.1042	0.0716	0.0446	0.0214	0.0000
0.2	0.0000	0.2354	0.2448	0.2123	0.1705	0.1303	0.0953	0.0658	0.0412	0.0197	0.0000
0.4	0.0000	0.1771	0.2042	0.1839	0.1506	0.1165	0.0858	0.0596	0.0374	0.0180	0.0000
0.6	0.0000	0.1333	0.1677	0.1567	0.1308	0.1023	0.0759	0.0529	0.0333	0.0160	0.0000
0.8	0.0000	0.1006	0.1355	0.1309	0.1112	0.0878	0.0656	0.0459	0.0290	0.0140	0.0000
1.0	0.0000	0.0756	0.1071	0.1065	0.0919	0.0732	0.0550	0.0387	0.0244	0.0118	0.0000
1.2	0.0000	0.0558	0.0820	0.0834	0.0729	0.0586	0.0442	0.0312	0.0197	0.0095	0.0000
1.4	0.0000	0.0393	0.0593	0.0615	0.0543	0.0439	0.0333	0.0235	0.0149	0.0072	0.0000
1.6	0.0000	0.0251	0.0386	0.0405	0.0360	0.0293	0.0222	0.0157	0.0100	0.0048	0.0000
1.8	0.0000	0.0122	0.0190	0.0201	0.0180	0.0146	0.0111	0.0079	0.0050	0.0024	0.0000
2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 10b. Time, pressure and acceleration for d = 1.0

ψ / λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
					time, t						
0.0	0.0000	2.6683	3.4385	4.0239	4.5277	4.9869	5.4193	5.8349	6.2401	6.6392	7.0355
0.2	0.0000	1.4666	2.2698	2.8798	3.4013	3.8732	4.3150	4.7378	5.1485	5.5522	5.9526
0.4	0.0000	1.2177	2.0312	2.6601	3.1968	3.6803	4.1307	4.5600	4.9759	5.3839	5.7882
0.6	0.0000	1.0731	1.8820	2.5243	3.0738	3.5675	4.0257	4.4609	4.8815	5.2934	5.7012
0.8	0.0000	0.9798	1.7771	2.4282	2.9881	3.4905	3.9555	4.3961	4.8208	5.2362	5.6472
1.0	0.0000	0.9168	1.7006	2.3569	2.9251	3.4348	3.9057	4.3507	4.7791	5.1975	5.6113
1.2	0.0000	0.8734	1.6446	2.3038	2.8782	3.3939	3.8695	4.3183	4.7497	5.1706	5.5867
1.4	0.0000	0.8438	1.6045	2.2651	2.8441	3.3642	3.8436	4.2954	4.7291	5.1521	5.5699
1.6	0.0000	0.8244	1.5776	2.2387	2.8207	3.3440	3.8261	4.2800	4.7155	5.1399	5.5590
1.8	0.0000	0.8135	1.5621	2.2233	2.8071	3.3323	3.8159	4.2712	4.7077	5.1329	5.5529
2.0	0.0000	0.8100	1.5570	2.2183	2.8026	3.3284	3.8126	4.2683	4.7051	5.1307	5.5509
					pressure, p						
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.2	0.1962	0.1647	0.1506	0.1415	0.1338	0.1272	0.1218	0.1177	0.1148	0.1131	0.1126
-0.4	0.3413	0.3180	0.2971	0.2809	0.2664	0.2537	0.2431	0.2351	0.2294	0.2261	0.2250
-0.6	0.4806	0.4637	0.4403	0.4184	0.3978	0.3793	0.3639	0.3520	0.3436	0.3387	0.3371
-0.8	0.6187	0.6054	0.5809	0.5543	0.5280	0.5040	0.4839	0.4683	0.4574	0.4510	0.4489
-1.0	0.7569	0.7455	0.7198	0.6889	0.6571	0.6278	0.6031	0.5840	0.5706	0.5628	0.5602
-1.2	0.8962	0.8852	0.8578	0.8224	0.7851	0.7506	0.7215	0.6990	0.6832	0.6739	0.6709
-1.4	1.0371	1.0256	0.9954	0.9550	0.9122	0.8724	0.8390	0.8132	0.7951	0.7845	0.7810
-1.6	1.1800	1.1672	1.1331	1.0870	1.0381	0.9931	0.9554	0.9264	0.9061	0.8943	0.8904
-1.8	1.3254	1.3106	1.2712	1.2184	1.1630	1.1125	1.0706	1.0386	1.0163	1.0032	0.9990
-2.0	1.4739	1.4562	1.4099	1.3491	1.2867	1.2306	1.1846	1.1496	1.1254	1.1113	1.1067
					horizontal acceleration						
0.0	0.0000	0.2646	0.2208	0.1721	0.1278	0.0916	0.0636	0.0422	0.0256	0.0121	0.0000
-0.2	0.0000	0.2117	0.1972	0.1626	0.1263	0.0938	0.0671	0.0455	0.0281	0.0134	0.0000
-0.4	0.0000	0.1639	0.1737	0.1523	0.1235	0.0950	0.0697	0.0482	0.0301	0.0145	0.0000
-0.6	0.0000	0.1267	0.1520	0.1418	0.1199	0.0952	0.0715	0.0504	0.0318	0.0154	0.0000
-0.8	0.0000	0.0999	0.1331	0.1318	0.1161	0.0949	0.0728	0.0520	0.0332	0.0161	0.0000
-1.0	0.0000	0.0809	0.1174	0.1229	0.1124	0.0942	0.0737	0.0533	0.0343	0.0168	0.0000
-1.2	0.0000	0.0677	0.1050	0.1153	0.1089	0.0934	0.0742	0.0543	0.0352	0.0173	0.0000
-1.4	0.0000	0.0587	0.0957	0.1093	0.1061	0.0927	0.0745	0.0550	0.0358	0.0176	0.0000
-1.6	0.0000	0.0529	0.0892	0.1050	0.1039	0.0920	0.0746	0.0554	0.0363	0.0179	0.0000
-1.8	0.0000	0.0496	0.0855	0.1024	0.1026	0.0916	0.0747	0.0557	0.0365	0.0180	0.0000
-2.0	0.0000	0.0485	0.0842	0.1015	0.1021	0.0915	0.0747	0.0558	0.0366	0.0181	0.0000
					vertical acceleration						
0.0	0.3268	0.0131	−0.0619	−0.0963	−0.1040	−0.0972	−0.0847	−0.0720	−0.0618	−0.0554	−0.0532
-0.2	0.2397	0.0592	−0.0304	−0.0717	−0.0852	−0.0833	−0.0746	−0.0646	−0.0561	−0.0506	−0.0487
-0.4	0.1880	0.0792	−0.0082	−0.0520	−0.0690	−0.0706	−0.0649	−0.0571	−0.0501	−0.0455	−0.0439
-0.6	0.1493	0.0814	0.0059	−0.0365	−0.0550	−0.0589	−0.0556	−0.0496	−0.0440	−0.0402	−0.0388
-0.8	0.1184	0.0745	0.0135	−0.0249	−0.0431	−0.0484	−0.0467	−0.0423	−0.0378	−0.0347	−0.0336
-1.0	0.0927	0.0636	0.0165	−0.0164	−0.0331	−0.0387	−0.0381	−0.0350	−0.0315	−0.0291	−0.0282
-1.2	0.0706	0.0511	0.0162	−0.0103	−0.0246	−0.0299	−0.0300	−0.0278	−0.0253	−0.0234	−0.0227
-1.4	0.0510	0.0382	0.0136	−0.0062	−0.0174	−0.0218	−0.0222	−0.0208	−0.0189	−0.0176	−0.0171
-1.6	0.0332	0.0254	0.0098	−0.0034	−0.0111	−0.0142	−0.0147	−0.0138	−0.0126	−0.0117	−0.0114
-1.8	0.0163	0.0126	0.0051	−0.0015	−0.0054	−0.0070	−0.0073	−0.0069	−0.0063	−0.0059	−0.0057
-2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE 10c. DISPLACEMENT AND VELOCITY NEAR THE SURFACE FOR $d = 1.0$

/λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ						horizontal displacement, x						
0.00	0.0000	0.0152	0.0242	0.0445	0.0707	0.0928	0.1306	0.2081	0.3326	0.4870	0.6225	5.8649
0.01	0.0000	0.0059	0.0117	0.0287	0.0544	0.0772	0.1165	0.1965	0.3233	0.4795	0.6160	5.8649
0.02	0.0000	0.0047	0.0094	0.0233	0.0458	0.0669	0.1054	0.1860	0.3145	0.4722	0.6096	5.8649
0.03	0.0000	0.0041	0.0082	0.0205	0.0407	0.0602	0.0969	0.1768	0.3062	0.4651	0.6034	5.8649
0.04	0.0000	0.0038	0.0075	0.0187	0.0373	0.0554	0.0904	0.1687	0.2983	0.4583	0.5973	5.8649
0.05	0.0000	0.0035	0.0070	0.0174	0.0348	0.0519	0.0852	0.1616	0.2909	0.4516	0.5913	5.8649
0.06	0.0000	0.0033	0.0066	0.0165	0.0329	0.0491	0.0810	0.1555	0.2840	0.4452	0.5855	5.8649
0.07	0.0000	0.0031	0.0063	0.0157	0.0313	0.0469	0.0775	0.1500	0.2776	0.4390	0.5798	5.8649
0.08	0.0000	0.0030	0.0060	0.0151	0.0301	0.0450	0.0745	0.1452	0.2716	0.4330	0.5743	5.8649
0.09	0.0000	0.0029	0.0058	0.0145	0.0290	0.0434	0.0720	0.1410	0.2659	0.4272	0.5688	5.8649
0.10	0.0000	0.0028	0.0056	0.0141	0.0281	0.0421	0.0698	0.1372	0.2607	0.4217	0.5635	5.8649
						vertical displacement, y						
0.00	−0.0652	−0.0564	−0.0513	−0.0396	−0.0246	−0.0121	0.0093	0.0523	0.1193	0.1989	0.2653	1.1438
0.01	0.0049	0.0051	0.0054	0.0077	0.0141	0.0218	0.0378	0.0749	0.1374	0.2142	0.2791	1.1517
0.02	0.0463	0.0464	0.0465	0.0475	0.0507	0.0554	0.0668	0.0980	0.1557	0.2295	0.2929	1.1597
0.03	0.0812	0.0812	0.0813	0.0819	0.0839	0.0869	0.0952	0.1212	0.1743	0.2450	0.3068	1.1676
0.04	0.1124	0.1124	0.1125	0.1129	0.1142	0.1164	0.1227	0.1444	0.1929	0.2606	0.3208	1.1755
0.05	0.1412	0.1412	0.1412	0.1415	0.1425	0.1442	0.1491	0.1674	0.2116	0.2763	0.3348	1.1835
0.06	0.1682	0.1682	0.1682	0.1684	0.1692	0.1706	0.1745	0.1901	0.2304	0.2920	0.3489	1.1914
0.07	0.1938	0.1938	0.1938	0.1940	0.1946	0.1957	0.1990	0.2124	0.2491	0.3078	0.3631	1.1994
0.08	0.2182	0.2182	0.2182	0.2184	0.2189	0.2199	0.2226	0.2343	0.2679	0.3237	0.3773	1.2073
0.09	0.2417	0.2418	0.2418	0.2419	0.2424	0.2431	0.2455	0.2558	0.2865	0.3396	0.3916	1.2153
0.10	0.2645	0.2645	0.2645	0.2646	0.2650	0.2657	0.2678	0.2768	0.3050	0.3555	0.4058	1.2233
						horizontal velocity, u						
0.00	0.0000	0.0929	0.1172	0.1591	0.2006	0.2298	0.2727	0.3443	0.4352	0.5263	0.5943	1.2599
0.01	0.2132	0.2136	0.2147	0.2216	0.2389	0.2573	0.2906	0.3541	0.4404	0.5291	0.5960	1.2596
0.02	0.2677	0.2678	0.2681	0.2705	0.2781	0.2885	0.3118	0.3655	0.4461	0.5322	0.5979	1.2593
0.03	0.3053	0.3054	0.3056	0.3068	0.3110	0.3174	0.3336	0.3779	0.4523	0.5355	0.5999	1.2589
0.04	0.3350	0.3350	0.3351	0.3359	0.3386	0.3428	0.3545	0.3908	0.4589	0.5389	0.6020	1.2586
0.05	0.3597	0.3598	0.3598	0.3604	0.3623	0.3653	0.3741	0.4039	0.4659	0.5426	0.6042	1.2583
0.06	0.3812	0.3812	0.3812	0.3816	0.3830	0.3853	0.3921	0.4169	0.4731	0.5464	0.6065	1.2580
0.07	0.4001	0.4001	0.4002	0.4005	0.4016	0.4034	0.4088	0.4296	0.4805	0.5504	0.6089	1.2576
0.08	0.4172	0.4172	0.4172	0.4175	0.4184	0.4198	0.4242	0.4419	0.4880	0.5545	0.6115	1.2573
0.09	0.4327	0.4327	0.4327	0.4330	0.4337	0.4349	0.4386	0.4538	0.4955	0.5587	0.6141	1.2570
0.10	0.4470	0.4470	0.4470	0.4472	0.4478	0.4488	0.4520	0.4651	0.5030	0.5630	0.6168	1.2567
						vertical velocity, v						
0.00	0.0000	0.0536	0.0674	0.0912	0.1143	0.1302	0.1530	0.1890	0.2300	0.2642	0.2843	0.0000
0.01	0.0000	0.0088	0.0174	0.0413	0.0725	0.0950	0.1261	0.1712	0.2185	0.2561	0.2779	0.0000
0.02	0.0000	0.0055	0.0109	0.0269	0.0513	0.0722	0.1047	0.1549	0.2074	0.2482	0.2716	0.0000
0.03	0.0000	0.0041	0.0083	0.0205	0.0401	0.0580	0.0885	0.1405	0.1968	0.2405	0.2653	0.0000
0.04	0.0000	0.0034	0.0067	0.0168	0.0331	0.0486	0.0764	0.1278	0.1867	0.2330	0.2592	0.0000
0.05	0.0000	0.0029	0.0058	0.0143	0.0284	0.0420	0.0672	0.1168	0.1772	0.2257	0.2532	0.0000
0.06	0.0000	0.0025	0.0050	0.0126	0.0250	0.0371	0.0600	0.1073	0.1683	0.2185	0.2473	0.0000
0.07	0.0000	0.0023	0.0045	0.0112	0.0224	0.0333	0.0542	0.0991	0.1600	0.2117	0.2416	0.0000
0.08	0.0000	0.0020	0.0041	0.0102	0.0203	0.0303	0.0495	0.0920	0.1522	0.2050	0.2359	0.0000
0.09	0.0000	0.0019	0.0037	0.0093	0.0186	0.0278	0.0456	0.0857	0.1450	0.1986	0.2304	0.0000
0.10	0.0000	0.0017	0.0035	0.0086	0.0172	0.0257	0.0423	0.0803	0.1383	0.1924	0.2250	0.0000

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TABLE 10*d*. TIME, PRESSURE AND ACCELERATION NEAR THE SURFACE FOR $d = 1.0$

/λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ						time, t						
0.00	0.0000	0.3277	0.4128	0.5602	0.7060	0.8084	0.9591	1.2104	1.5295	1.8508	2.0926	7.0355
0.01	0.0000	0.0276	0.0549	0.1329	0.2449	0.3366	0.4804	0.7288	1.0482	1.3706	1.6132	6.5649
0.02	0.0000	0.0175	0.0350	0.0868	0.1687	0.2435	0.3720	0.6106	0.9275	1.2501	1.4933	6.4535
0.03	0.0000	0.0135	0.0269	0.0671	0.1323	0.1945	0.3076	0.5327	0.8450	1.1670	1.4107	6.3790
0.04	0.0000	0.0112	0.0224	0.0558	0.1108	0.1642	0.2645	0.4753	0.7811	1.1019	1.3457	6.3220
0.05	0.0000	0.0097	0.0194	0.0485	0.0964	0.1435	0.2336	0.4307	0.7288	1.0478	1.2916	6.2755
0.06	0.0000	0.0086	0.0173	0.0432	0.0861	0.1284	0.2104	0.3950	0.6847	1.0013	1.2448	6.2360
0.07	0.0000	0.0078	0.0157	0.0392	0.0782	0.1168	0.1922	0.3656	0.6468	0.9604	1.2034	6.2017
0.08	0.0000	0.0072	0.0144	0.0361	0.0720	0.1077	0.1776	0.3412	0.6137	0.9240	1.1663	6.1714
0.09	0.0000	0.0067	0.0134	0.0335	0.0670	0.1002	0.1657	0.3205	0.5845	0.8911	1.1327	6.1442
0.10	0.0000	0.0063	0.0126	0.0314	0.0628	0.0940	0.1556	0.3027	0.5585	0.8612	1.1018	6.1196
						pressure, p						
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.01	0.0233	0.0233	0.0232	0.0225	0.0209	0.0195	0.0175	0.0146	0.0122	0.0106	0.0098	0.0056
0.02	0.0374	0.0374	0.0374	0.0371	0.0361	0.0349	0.0326	0.0283	0.0241	0.0211	0.0195	0.0113
0.03	0.0495	0.0495	0.0495	0.0493	0.0487	0.0478	0.0458	0.0411	0.0356	0.0314	0.0291	0.0169
0.04	0.0605	0.0605	0.0605	0.0604	0.0599	0.0593	0.0576	0.0531	0.0467	0.0416	0.0386	0.0225
0.05	0.0708	0.0708	0.0708	0.0707	0.0704	0.0699	0.0685	0.0643	0.0575	0.0515	0.0480	0.0282
0.06	0.0806	0.0806	0.0806	0.0805	0.0802	0.0799	0.0787	0.0749	0.0680	0.0614	0.0574	0.0338
0.07	0.0900	0.0900	0.0900	0.0899	0.0897	0.0894	0.0884	0.0850	0.0782	0.0711	0.0666	0.0394
0.08	0.0991	0.0991	0.0991	0.0990	0.0988	0.0986	0.0978	0.0947	0.0880	0.0806	0.0758	0.0451
0.09	0.1079	0.1079	0.1079	0.1079	0.1077	0.1075	0.1068	0.1041	0.0976	0.0900	0.0848	0.0507
0.10	0.1165	0.1165	0.1165	0.1165	0.1164	0.1162	0.1156	0.1132	0.1070	0.0992	0.0938	0.0563
						horizontal acceleration						
0.00	0.0000	0.2842	0.2845	0.2848	0.2849	0.2850	0.2850	0.2849	0.2844	0.2826	0.2796	0.0000
0.01	0.0000	0.0272	0.0533	0.1192	0.1841	0.2159	0.2438	0.2644	0.2738	0.2761	0.2749	0.0000
0.02	0.0000	0.0136	0.0270	0.0654	0.1186	0.1568	0.2016	0.2425	0.2627	0.2696	0.2701	0.0000
0.03	0.0000	0.0090	0.0180	0.0443	0.0844	0.1180	0.1658	0.2205	0.2513	0.2628	0.2652	0.0000
0.04	0.0000	0.0067	0.0134	0.0333	0.0648	0.0930	0.1380	0.1996	0.2398	0.2560	0.2603	0.0000
0.05	0.0000	0.0054	0.0107	0.0266	0.0523	0.0761	0.1169	0.1805	0.2284	0.2492	0.2553	0.0000
0.06	0.0000	0.0044	0.0089	0.0221	0.0437	0.0642	0.1006	0.1635	0.2171	0.2423	0.2503	0.0000
0.07	0.0000	0.0038	0.0076	0.0189	0.0374	0.0553	0.0880	0.1486	0.2062	0.2354	0.2453	0.0000
0.08	0.0000	0.0033	0.0066	0.0165	0.0327	0.0485	0.0779	0.1355	0.1957	0.2285	0.2403	0.0000
0.09	0.0000	0.0029	0.0058	0.0146	0.0290	0.0431	0.0697	0.1242	0.1857	0.2218	0.2353	0.0000
0.10	0.0000	0.0026	0.0052	0.0131	0.0260	0.0387	0.0630	0.1143	0.1763	0.2151	0.2304	0.0000
						vertical acceleration						
0.00	0.3268	0.1630	0.1622	0.1600	0.1567	0.1538	0.1487	0.1375	0.1184	0.0937	0.0719	−0.0532
0.01	0.3193	0.3181	0.3148	0.2960	0.2598	0.2330	0.2010	0.1649	0.1322	0.1015	0.0774	−0.0530
0.02	0.3127	0.3124	0.3115	0.3056	0.2888	0.2695	0.2362	0.1883	0.1449	0.1090	0.0828	−0.0528
0.03	0.3068	0.3067	0.3063	0.3035	0.2947	0.2826	0.2564	0.2070	0.1566	0.1161	0.0879	−0.0525
0.04	0.3014	0.3014	0.3011	0.2995	0.2942	0.2863	0.2667	0.2212	0.1670	0.1228	0.0928	−0.0523
0.05	0.2964	0.2963	0.2962	0.2951	0.2916	0.2861	0.2714	0.2314	0.1761	0.1291	0.0975	−0.0521
0.06	0.2916	0.2916	0.2915	0.2907	0.2882	0.2842	0.2730	0.2386	0.1840	0.1349	0.1020	−0.0519
0.07	0.2871	0.2870	0.2870	0.2864	0.2845	0.2815	0.2727	0.2433	0.1907	0.1403	0.1062	−0.0517
0.08	0.2827	0.2827	0.2826	0.2822	0.2807	0.2783	0.2713	0.2461	0.1963	0.1452	0.1102	−0.0514
0.09	0.2785	0.2785	0.2785	0.2781	0.2769	0.2750	0.2692	0.2476	0.2009	0.1497	0.1140	−0.0512
0.10	0.2745	0.2745	0.2744	0.2741	0.2732	0.2716	0.2668	0.2481	0.2046	0.1538	0.1175	−0.0510

TABLE 10*e*. MEAN DEPTH AS A FUNCTION OF ψ FOR $d = 1.0$

ψ = 0(0.01)0.1	1.9411	1.9295	1.9185	1.9076	1.8968	1.8861	1.8754	1.8648	1.8543	1.8438	1.8334
ψ = 0(0.2)2.0	1.9411	1.7306	1.5308	1.3350	1.1415	0.9496	0.7587	0.5685	0.3788	0.1893	0.0000

TABLE 11*a*. DISPLACEMENT AND VELOCITY FOR $d = 2.0$

($1/2F^2 = 1.037315$, $h = 1.94082$, $L = 5.91907$, $\bar{y}_s = 0.54120$.)

ψ \ λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
					horizontal displacement, x						
0.0	0.0000	0.5372	0.8751	1.1728	1.4501	1.7150	1.9718	2.2229	2.4704	2.7155	2.9595
0.2	0.0000	0.4496	0.8066	1.1189	1.4076	1.6818	1.9464	2.2046	2.4585	2.7096	2.9595
0.4	0.0000	0.3995	0.7548	1.0742	1.3707	1.6523	1.9237	2.1880	2.4477	2.7043	2.9595
0.6	0.0000	0.3704	0.7173	1.0384	1.3398	1.6268	1.9037	2.1734	2.4380	2.6995	2.9595
0.8	0.0000	0.3522	0.6905	1.0107	1.3145	1.6055	1.8867	2.1607	2.4297	2.6954	2.9595
1.0	0.0000	0.3401	0.6714	0.9895	1.2945	1.5881	1.8726	2.1501	2.4226	2.6919	2.9595
1.2	0.0000	0.3320	0.6578	0.9738	1.2790	1.5743	1.8612	2.1415	2.4169	2.6890	2.9595
1.4	0.0000	0.3265	0.6483	0.9625	1.2676	1.5640	1.8526	2.1349	2.4124	2.6868	2.9595
1.6	0.0000	0.3229	0.6421	0.9549	1.2598	1.5568	1.8466	2.1302	2.4093	2.6852	2.9595
1.8	0.0000	0.3209	0.6386	0.9505	1.2552	1.5526	1.8430	2.1275	2.4074	2.6842	2.9595
2.0	0.0000	0.3203	0.6374	0.9491	1.2538	1.5512	1.8418	2.1265	2.4068	2.6839	2.9595
					vertical displacement, y						
0.0	−0.0035	0.2776	0.4206	0.5254	0.6060	0.6687	0.7169	0.7526	0.7772	0.7917	0.7964
0.2	0.4528	0.5248	0.6256	0.7115	0.7814	0.8372	0.8807	0.9133	0.9359	0.9492	0.9536
0.4	0.7373	0.7711	0.8375	0.9042	0.9625	1.0107	1.0490	1.0780	1.0983	1.1103	1.1142
0.6	0.9869	1.0061	1.0503	1.1006	1.1477	1.1881	1.2210	1.2463	1.2641	1.2746	1.2781
0.8	1.2190	1.2311	1.2612	1.2986	1.3357	1.3687	1.3962	1.4176	1.4328	1.4418	1.4448
1.0	1.4405	1.4485	1.4695	1.4969	1.5254	1.5516	1.5738	1.5914	1.6039	1.6115	1.6140
1.2	1.6552	1.6606	1.6751	1.6948	1.7161	1.7361	1.7534	1.7672	1.7772	1.7832	1.7852
1.4	1.8653	1.8689	1.8787	1.8923	1.9073	1.9217	1.9344	1.9446	1.9520	1.9565	1.9580
1.6	2.0724	2.0746	2.0806	2.0892	2.0988	2.1081	2.1164	2.1231	2.1280	2.1310	2.1320
1.8	2.2776	2.2787	2.2816	2.2857	2.2904	2.2950	2.2990	2.3024	2.3048	2.3063	2.3068
2.0	2.4820	2.4820	2.4820	2.4820	2.4820	2.4820	2.4820	2.4820	2.4820	2.4820	2.4820
					horizontal velocity, u						
0.0	0.0000	0.6932	0.8755	0.9978	1.0877	1.1555	1.2064	1.2435	1.2687	1.2834	1.2883
0.2	0.6344	0.7468	0.8815	0.9862	1.0675	1.1306	1.1789	1.2146	1.2391	1.2535	1.2582
0.4	0.7618	0.8109	0.9001	0.9841	1.0546	1.1116	1.1564	1.1901	1.2136	1.2274	1.2320
0.6	0.8359	0.8628	0.9224	0.9876	1.0471	1.0976	1.1385	1.1699	1.1920	1.2051	1.2095
0.8	0.8849	0.9018	0.9433	0.9938	1.0434	1.0876	1.1244	1.1533	1.1740	1.1864	1.1905
1.0	0.9190	0.9307	0.9610	1.0007	1.0420	1.0805	1.1136	1.1401	1.1594	1.1710	1.1749
1.2	0.9431	0.9517	0.9750	1.0070	1.0419	1.0757	1.1055	1.1299	1.1478	1.1587	1.1624
1.4	0.9598	0.9666	0.9854	1.0121	1.0424	1.0725	1.0997	1.1223	1.1391	1.1494	1.1529
1.6	0.9709	0.9766	0.9926	1.0159	1.0430	1.0705	1.0958	1.1171	1.1330	1.1429	1.1462
1.8	0.9772	0.9824	0.9968	1.0182	1.0434	1.0694	1.0935	1.1140	1.1294	1.1390	1.1423
2.0	0.9793	0.9842	0.9982	1.0190	1.0436	1.0690	1.0928	1.1130	1.1283	1.1378	1.1410
					vertical velocity, v						
0.0	0.0000	0.3205	0.3368	0.3190	0.2856	0.2441	0.1981	0.1497	0.1002	0.0502	0.0000
0.2	0.0000	0.1968	0.2524	0.2565	0.2380	0.2078	0.1709	0.1303	0.0877	0.0441	0.0000
0.4	0.0000	0.1255	0.1862	0.2026	0.1950	0.1740	0.1452	0.1117	0.0756	0.0381	0.0000
0.6	0.0000	0.0849	0.1372	0.1578	0.1572	0.1433	0.1212	0.0942	0.0641	0.0324	0.0000
0.8	0.0000	0.0597	0.1014	0.1215	0.1246	0.1158	0.0992	0.0778	0.0533	0.0270	0.0000
1.0	0.0000	0.0426	0.0747	0.0922	0.0967	0.0914	0.0792	0.0626	0.0431	0.0219	0.0000
1.2	0.0000	0.0303	0.0541	0.0681	0.0727	0.0697	0.0610	0.0485	0.0336	0.0171	0.0000
1.4	0.0000	0.0208	0.0376	0.0480	0.0519	0.0503	0.0444	0.0355	0.0246	0.0126	0.0000
1.6	0.0000	0.0130	0.0237	0.0306	0.0334	0.0326	0.0289	0.0232	0.0161	0.0083	0.0000
1.8	0.0000	0.0063	0.0115	0.0149	0.0163	0.0160	0.0142	0.0115	0.0080	0.0041	0.0000
2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 11*b*. Time, pressure and acceleration for $d = 2.0$

λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
ψ						time, t					
0.0	0.0000	1.5467	1.9768	2.2943	2.5600	2.7959	3.0131	3.2180	3.4149	3.6069	3.7965
0.2	0.0000	0.6663	1.1055	1.4398	1.7207	1.9700	2.1990	2.4146	2.6214	2.8228	3.0217
0.4	0.0000	0.5131	0.9294	1.2685	1.5593	1.8191	2.0583	2.2835	2.4993	2.7095	2.9169
0.6	0.0000	0.4383	0.8278	1.1643	1.4604	1.7280	1.9756	2.2090	2.4330	2.6511	2.8663
0.8	0.0000	0.3954	0.7628	1.0936	1.3919	1.6649	1.9191	2.1596	2.3906	2.6156	2.8377
1.0	0.0000	0.3686	0.7193	1.0438	1.3425	1.6191	1.8783	2.1245	2.3614	2.5924	2.8205
1.2	0.0000	0.3510	0.6896	1.0086	1.3066	1.5855	1.8486	2.0992	2.3409	2.5767	2.8097
1.4	0.0000	0.3393	0.6694	0.9841	1.2812	1.5615	1.8272	2.0812	2.3266	2.5662	2.8030
1.6	0.0000	0.3319	0.6564	0.9680	1.2643	1.5453	1.8128	2.0691	2.3171	2.5594	2.7990
1.8	0.0000	0.3278	0.6491	0.9588	1.2545	1.5360	1.8045	2.0622	2.3116	2.5556	2.7968
2.0	0.0000	0.3265	0.6467	0.9558	1.2514	1.5329	1.8018	2.0599	2.3099	2.5544	2.7962
						pressure, p					
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.2721	0.2498	0.2322	0.2225	0.2161	0.2114	0.2078	0.2049	0.2029	0.2016	0.2012
0.4	0.4782	0.4669	0.4499	0.4369	0.4270	0.4190	0.4126	0.4074	0.4036	0.4013	0.4005
0.6	0.6780	0.6715	0.6583	0.6452	0.6336	0.6235	0.6148	0.6077	0.6024	0.5991	0.5980
0.8	0.8766	0.8722	0.8619	0.8495	0.8370	0.8253	0.8148	0.8060	0.7993	0.7951	0.7937
1.0	1.0757	1.0723	1.0634	1.0515	1.0383	1.0252	1.0130	1.0025	0.9944	0.9894	0.9877
1.2	1.2759	1.2729	1.2645	1.2524	1.2383	1.2235	1.2095	1.1973	1.1878	1.1819	1.1798
1.4	1.4779	1.4749	1.4662	1.4532	1.4375	1.4207	1.4046	1.3904	1.3794	1.3725	1.3701
1.6	1.6820	1.6787	1.6690	1.6543	1.6363	1.6170	1.5982	1.5818	1.5691	1.5610	1.5583
1.8	1.8888	1.8848	1.8735	1.8561	1.8350	1.8124	1.7905	1.7714	1.7566	1.7473	1.7441
2.0	2.0988	2.0939	2.0801	2.0591	2.0338	2.0069	1.9812	1.9589	1.9418	1.9310	1.9274
						horizontal acceleration					
0.0	0.0000	0.4377	0.4053	0.3621	0.3129	0.2606	0.2074	0.1544	0.1022	0.0509	0.0000
0.2	0.0000	0.2824	0.3181	0.3038	0.2722	0.2324	0.1883	0.1422	0.0950	0.0476	0.0000
0.4	0.0000	0.1764	0.2407	0.2489	0.2328	0.2044	0.1690	0.1294	0.0874	0.0440	0.0000
0.6	0.0000	0.1173	0.1811	0.2015	0.1971	0.1783	0.1505	0.1170	0.0798	0.0404	0.0000
0.8	0.0000	0.0831	0.1382	0.1632	0.1664	0.1549	0.1336	0.1054	0.0727	0.0370	0.0000
1.0	0.0000	0.0620	0.1081	0.1335	0.1411	0.1351	0.1188	0.0952	0.0663	0.0339	0.0000
1.2	0.0000	0.0484	0.0871	0.1113	0.1213	0.1189	0.1066	0.0865	0.0608	0.0313	0.0000
1.4	0.0000	0.0397	0.0728	0.0954	0.1064	0.1065	0.0970	0.0796	0.0564	0.0292	0.0000
1.6	0.0000	0.0341	0.0635	0.0848	0.0962	0.0977	0.0901	0.0747	0.0532	0.0276	0.0000
1.8	0.0000	0.0311	0.0583	0.0786	0.0902	0.0925	0.0860	0.0717	0.0513	0.0267	0.0000
2.0	0.0000	0.0301	0.0566	0.0766	0.0883	0.0908	0.0846	0.0707	0.0506	0.0264	0.0000
						vertical acceleration					
0.0	0.5158	0.0904	−0.0161	−0.0953	−0.1540	−0.1964	−0.2257	−0.2451	−0.2570	−0.2633	−0.2653
0.2	0.3588	0.1968	0.0592	−0.0323	−0.0973	−0.1438	−0.1765	−0.1985	−0.2124	−0.2201	−0.2225
0.4	0.2699	0.1986	0.0915	0.0065	−0.0568	−0.1032	−0.1364	−0.1592	−0.1739	−0.1821	−0.1847
0.6	0.2065	0.1694	0.0962	0.0265	−0.0297	−0.0727	−0.1042	−0.1263	−0.1408	−0.1489	−0.1516
0.8	0.1583	0.1366	0.0874	0.0339	−0.0129	−0.0503	−0.0786	−0.0989	−0.1123	−0.1200	−0.1225
1.0	0.1203	0.1067	0.0734	0.0338	−0.0033	−0.0343	−0.0584	−0.0760	−0.0878	−0.0946	−0.0968
1.2	0.0893	0.0805	0.0580	0.0295	0.0015	−0.0229	−0.0423	−0.0567	−0.0666	−0.0722	−0.0741
1.4	0.0632	0.0576	0.0427	0.0232	0.0032	−0.0147	−0.0293	−0.0402	−0.0478	−0.0522	−0.0536
1.6	0.0404	0.0371	0.0280	0.0158	0.0030	−0.0087	−0.0184	−0.0257	−0.0308	−0.0338	−0.0348
1.8	0.0197	0.0181	0.0138	0.0080	0.0017	−0.0041	−0.0089	−0.0126	−0.0151	−0.0166	−0.0171
2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE 11c. DISPLACEMENT AND VELOCITY NEAR THE SURFACE FOR $d = 2.0$

λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ						horizontal displacement, x						
0.00	0.0000	0.0082	0.0131	0.0241	0.0382	0.0501	0.0705	0.1123	0.1790	0.2614	0.3333	2.9595
0.01	0.0000	0.0025	0.0051	0.0126	0.0247	0.0361	0.0568	0.1002	0.1690	0.2531	0.3261	2.9595
0.02	0.0000	0.0020	0.0040	0.0101	0.0200	0.0298	0.0486	0.0907	0.1601	0.2454	0.3191	2.9595
0.03	0.0000	0.0018	0.0035	0.0088	0.0176	0.0264	0.0435	0.0834	0.1522	0.2381	0.3124	2.9595
0.04	0.0000	0.0016	0.0032	0.0081	0.0161	0.0241	0.0399	0.0778	0.1453	0.2312	0.3061	2.9595
0.05	0.0000	0.0015	0.0030	0.0075	0.0150	0.0225	0.0374	0.0734	0.1393	0.2249	0.3000	2.9595
0.06	0.0000	0.0014	0.0028	0.0071	0.0142	0.0213	0.0353	0.0698	0.1340	0.2191	0.2942	2.9595
0.07	0.0000	0.0014	0.0027	0.0068	0.0135	0.0203	0.0337	0.0668	0.1294	0.2136	0.2888	2.9595
0.08	0.0000	0.0013	0.0026	0.0065	0.0130	0.0195	0.0324	0.0643	0.1253	0.2086	0.2836	2.9595
0.09	0.0000	0.0013	0.0025	0.0063	0.0125	0.0188	0.0313	0.0621	0.1217	0.2040	0.2787	2.9595
0.10	0.0000	0.0012	0.0024	0.0061	0.0121	0.0182	0.0303	0.0603	0.1184	0.1997	0.2740	2.9595
						vertical displacement, y						
0.00	−0.0035	0.0012	0.0040	0.0104	0.0185	0.0253	0.0369	0.0603	0.0971	0.1412	0.1785	0.7964
0.01	0.0567	0.0568	0.0568	0.0574	0.0591	0.0616	0.0679	0.0849	0.1166	0.1575	0.1931	0.8042
0.02	0.0923	0.0923	0.0924	0.0926	0.0933	0.0945	0.0979	0.1098	0.1365	0.1740	0.2078	0.8120
0.03	0.1223	0.1223	0.1223	0.1224	0.1229	0.1236	0.1258	0.1343	0.1565	0.1907	0.2227	0.8198
0.04	0.1492	0.1492	0.1492	0.1493	0.1496	0.1501	0.1516	0.1580	0.1765	0.2075	0.2377	0.8276
0.05	0.1739	0.1739	0.1739	0.1740	0.1742	0.1746	0.1758	0.1807	0.1963	0.2244	0.2528	0.8354
0.06	0.1972	0.1972	0.1972	0.1972	0.1974	0.1977	0.1986	0.2026	0.2159	0.2413	0.2679	0.8432
0.07	0.2192	0.2192	0.2192	0.2193	0.2194	0.2197	0.2204	0.2237	0.2351	0.2581	0.2831	0.8510
0.08	0.2403	0.2403	0.2403	0.2404	0.2405	0.2407	0.2413	0.2441	0.2540	0.2749	0.2983	0.8589
0.09	0.2606	0.2606	0.2606	0.2607	0.2608	0.2609	0.2615	0.2639	0.2726	0.2915	0.3134	0.8667
0.10	0.2803	0.2803	0.2803	0.2803	0.2804	0.2805	0.2810	0.2831	0.2908	0.3081	0.3286	0.8745
						horizontal velocity, u						
0.00	0.0000	0.0859	0.1083	0.1470	0.1854	0.2123	0.2519	0.3178	0.4013	0.4850	0.5474	1.2883
0.01	0.2482	0.2483	0.2486	0.2508	0.2579	0.2674	0.2889	0.3383	0.4125	0.4915	0.5519	1.2867
0.02	0.3113	0.3113	0.3114	0.3121	0.3146	0.3185	0.3293	0.3627	0.4254	0.4988	0.5568	1.2851
0.03	0.3549	0.3549	0.3549	0.3553	0.3566	0.3587	0.3650	0.3878	0.4395	0.5068	0.5621	1.2835
0.04	0.3891	0.3891	0.3892	0.3894	0.3902	0.3915	0.3956	0.4119	0.4542	0.5154	0.5678	1.2820
0.05	0.4177	0.4177	0.4177	0.4179	0.4184	0.4193	0.4222	0.4344	0.4692	0.5243	0.5739	1.2804
0.06	0.4423	0.4423	0.4423	0.4424	0.4429	0.4436	0.4457	0.4551	0.4840	0.5336	0.5801	1.2789
0.07	0.4641	0.4641	0.4641	0.4642	0.4645	0.4651	0.4667	0.4742	0.4985	0.5430	0.5866	1.2773
0.08	0.4836	0.4836	0.4837	0.4837	0.4840	0.4844	0.4858	0.4919	0.5125	0.5525	0.5933	1.2758
0.09	0.5014	0.5014	0.5014	0.5015	0.5017	0.5021	0.5032	0.5083	0.5260	0.5619	0.6001	1.2743
0.10	0.5178	0.5178	0.5178	0.5178	0.5180	0.5183	0.5193	0.5236	0.5389	0.5713	0.6069	1.2728
						vertical velocity, v						
0.00	0.0000	0.0496	0.0624	0.0845	0.1061	0.1210	0.1427	0.1774	0.2184	0.2551	0.2791	0.0000
0.01	0.0000	0.0051	0.0102	0.0252	0.0480	0.0676	0.0981	0.1460	0.1976	0.2405	0.2674	0.0000
0.02	0.0000	0.0032	0.0063	0.0158	0.0312	0.0458	0.0720	0.1210	0.1786	0.2265	0.2560	0.0000
0.03	0.0000	0.0024	0.0048	0.0119	0.0237	0.0352	0.0569	0.1022	0.1618	0.2132	0.2451	0.0000
0.04	0.0000	0.0019	0.0039	0.0097	0.0194	0.0289	0.0473	0.0881	0.1470	0.2008	0.2345	0.0000
0.05	0.0000	0.0017	0.0033	0.0083	0.0165	0.0247	0.0406	0.0773	0.1341	0.1892	0.2244	0.0000
0.06	0.0000	0.0014	0.0029	0.0072	0.0145	0.0216	0.0358	0.0689	0.1231	0.1784	0.2148	0.0000
0.07	0.0000	0.0013	0.0026	0.0065	0.0129	0.0193	0.0320	0.0622	0.1135	0.1684	0.2057	0.0000
0.08	0.0000	0.0012	0.0023	0.0058	0.0117	0.0175	0.0290	0.0567	0.1052	0.1593	0.1970	0.0000
0.09	0.0000	0.0011	0.0021	0.0053	0.0107	0.0160	0.0266	0.0522	0.0979	0.1509	0.1888	0.0000
0.10	0.0000	0.0010	0.0020	0.0049	0.0098	0.0148	0.0245	0.0483	0.0915	0.1431	0.1810	0.0000

Table 11*d*. Time, pressure and acceleration near the surface for $d = 2.0$

λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ						time, t						
0.00	0.0000	0.1918	0.2416	0.3278	0.4130	0.4728	0.5608	0.7072	0.8928	1.0788	1.2181	3.7965
0.01	0.0000	0.0102	0.0204	0.0505	0.0981	0.1416	0.2163	0.3548	0.5385	0.7247	0.8645	3.4515
0.02	0.0000	0.0065	0.0130	0.0323	0.0642	0.0951	0.1531	0.2751	0.4517	0.6363	0.7761	3.3710
0.03	0.0000	0.0050	0.0100	0.0249	0.0497	0.0741	0.1213	0.2277	0.3945	0.5762	0.7154	3.3175
0.04	0.0000	0.0041	0.0083	0.0207	0.0414	0.0619	0.1021	0.1960	0.3524	0.5299	0.6681	3.2768
0.05	0.0000	0.0036	0.0072	0.0180	0.0360	0.0538	0.0891	0.1733	0.3196	0.4923	0.6291	3.2438
0.06	0.0000	0.0032	0.0064	0.0161	0.0321	0.0480	0.0797	0.1562	0.2934	0.4608	0.5959	3.2159
0.07	0.0000	0.0029	0.0058	0.0146	0.0291	0.0437	0.0725	0.1429	0.2718	0.4340	0.5671	3.1919
0.08	0.0000	0.0027	0.0054	0.0134	0.0268	0.0402	0.0669	0.1322	0.2539	0.4107	0.5416	3.1707
0.09	0.0000	0.0025	0.0050	0.0125	0.0250	0.0374	0.0623	0.1234	0.2387	0.3903	0.5189	3.1518
0.10	0.0000	0.0023	0.0047	0.0117	0.0234	0.0351	0.0584	0.1160	0.2256	0.3723	0.4986	3.1347
						pressure, p						
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.01	0.0317	0.0317	0.0317	0.0314	0.0306	0.0296	0.0275	0.0238	0.0200	0.0173	0.0159	0.0101
0.02	0.0510	0.0510	0.0510	0.0508	0.0505	0.0499	0.0484	0.0445	0.0388	0.0341	0.0314	0.0202
0.03	0.0675	0.0675	0.0675	0.0675	0.0673	0.0669	0.0659	0.0625	0.0563	0.0503	0.0466	0.0303
0.04	0.0827	0.0827	0.0827	0.0826	0.0825	0.0822	0.0815	0.0788	0.0728	0.0660	0.0615	0.0404
0.05	0.0969	0.0968	0.0968	0.0968	0.0967	0.0965	0.0960	0.0938	0.0882	0.0811	0.0760	0.0505
0.06	0.1104	0.1104	0.1104	0.1103	0.1103	0.1101	0.1097	0.1079	0.1029	0.0957	0.0902	0.0606
0.07	0.1234	0.1234	0.1234	0.1234	0.1233	0.1232	0.1228	0.1213	0.1168	0.1098	0.1041	0.0707
0.08	0.1360	0.1360	0.1360	0.1360	0.1359	0.1358	0.1355	0.1343	0.1303	0.1235	0.1177	0.0807
0.09	0.1483	0.1483	0.1483	0.1483	0.1482	0.1482	0.1479	0.1468	0.1433	0.1368	0.1309	0.0908
0.10	0.1603	0.1603	0.1603	0.1603	0.1603	0.1602	0.1600	0.1591	0.1559	0.1498	0.1440	0.1009
						horizontal acceleration						
0.00	0.0000	0.4490	0.4494	0.4499	0.4502	0.4502	0.4502	0.4500	0.4499	0.4492	0.4477	0.0000
0.01	0.0000	0.0215	0.0429	0.1038	0.1882	0.2487	0.3197	0.3846	0.4172	0.4299	0.4336	0.0000
0.02	0.0000	0.0107	0.0214	0.0530	0.1031	0.1481	0.2198	0.3177	0.3822	0.4096	0.4191	0.0000
0.03	0.0000	0.0071	0.0142	0.0353	0.0698	0.1025	0.1607	0.2611	0.3471	0.3888	0.4043	0.0000
0.04	0.0000	0.0053	0.0106	0.0264	0.0524	0.0777	0.1248	0.2172	0.3140	0.3680	0.3892	0.0000
0.05	0.0000	0.0042	0.0084	0.0210	0.0418	0.0623	0.1013	0.1837	0.2837	0.3474	0.3741	0.0000
0.06	0.0000	0.0035	0.0070	0.0174	0.0347	0.0518	0.0849	0.1580	0.2567	0.3275	0.3592	0.0000
0.07	0.0000	0.0030	0.0060	0.0149	0.0297	0.0443	0.0729	0.1380	0.2331	0.3084	0.3445	0.0000
0.08	0.0000	0.0026	0.0052	0.0129	0.0258	0.0386	0.0638	0.1221	0.2124	0.2904	0.3301	0.0000
0.09	0.0000	0.0023	0.0046	0.0114	0.0229	0.0342	0.0566	0.1092	0.1945	0.2735	0.3161	0.0000
0.10	0.0000	0.0021	0.0041	0.0102	0.0205	0.0306	0.0507	0.0986	0.1788	0.2577	0.3026	0.0000
						vertical acceleration						
0.00	0.5158	0.2582	0.2574	0.2550	0.2515	0.2484	0.2428	0.2308	0.2104	0.1835	0.1593	−0.2653
0.01	0.5021	0.5017	0.5003	0.4912	0.4650	0.4350	0.3841	0.3129	0.2533	0.2082	0.1765	−0.2630
0.02	0.4899	0.4898	0.4894	0.4870	0.4787	0.4665	0.4365	0.3677	0.2897	0.2307	0.1927	−0.2608
0.03	0.4792	0.4791	0.4789	0.4778	0.4739	0.4678	0.4506	0.3985	0.3186	0.2508	0.2077	−0.2585
0.04	0.4693	0.4693	0.4692	0.4686	0.4663	0.4626	0.4518	0.4138	0.3402	0.2683	0.2213	−0.2563
0.05	0.4601	0.4601	0.4601	0.4596	0.4582	0.4558	0.4484	0.4202	0.3557	0.2832	0.2337	−0.2541
0.06	0.4515	0.4515	0.4514	0.4511	0.4501	0.4484	0.4431	0.4216	0.3662	0.2956	0.2447	−0.2519
0.07	0.4432	0.4432	0.4432	0.4430	0.4422	0.4409	0.4369	0.4201	0.3728	0.3057	0.2543	−0.2497
0.08	0.4354	0.4354	0.4353	0.4352	0.4346	0.4336	0.4305	0.4170	0.3765	0.3136	0.2627	−0.2475
0.09	0.4278	0.4278	0.4278	0.4277	0.4272	0.4264	0.4239	0.4129	0.3781	0.3198	0.2698	−0.2454
0.10	0.4205	0.4205	0.4205	0.4204	0.4200	0.4194	0.4173	0.4082	0.3781	0.3244	0.2758	−0.2432

Table 11*e*. Mean depth as a function of ψ for $d = 2.0$

$\psi = 0(0.01)0.1$	1.9408	1.9286	1.9172	1.9060	1.8949	1.8839	1.8730	1.8622	1.8514	1.8407	1.8301
$\psi = 0(0.2)2.0$	1.9408	1.7263	1.5262	1.3309	1.1383	0.9472	0.7570	0.5673	0.3780	0.1890	0.0000

Table 12a. Displacement and velocity for d = 10.0 (also applicable, with digits in parenthesis, to the deep-water wave)

($1/2F^2$ = 5.000000, h = 1.89781, L = 1.18482, $\bar{y}_s$ = 0.20219.)

/λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
ψ						horizontal displacement, x					
0.0	0.0000	0.1085	0.1763	0.2360	0.2914	0.3443	0.3955	0.4456	0.4949	0.5438	0.5924
0.2	0.0000	0.0675	0.1333	0.1966	0.2573	0.3160	0.3731	0.4289	0.4838	0.5383	0.5924
0.4	0.0000	0.0618	0.1232	0.1841	0.2442	0.3036	0.3623	0.4204	0.4780	0.5353	0.5924
0.6	0.0000	0.0601	0.1201	0.1800	0.2396	0.2989	0.3580	0.4169	0.4755	0.5340	0.5924
0.8	0.0000	0.0596	0.1191	0.1785	0.2379	0.2972	0.3564	0.4155	0.4745	0.5335	0.5924
1.0	0.0000	0.0594	0.1187	0.1780	0.2373	0.2966	0.3558	0.4150	0.4741	0.5333	0.5924
1.2	0.0000	0.0593	0.1186	0.1778	0.2371	0.2963	0.3556	0.4148	0.4740	0.5332	0.5924
1.4	0.0000	0.0593	0.1185	0.1778	0.2370	0.2963	0.3555	0.4147	0.4740	0.5332	0.5924
1.6	0.0000	0.0592	0.1185	0.1777	0.2370	0.2962	0.3555	0.4147	0.4739	0.5332	0.5924
1.8	0.0000	0.0592	0.1185	0.1777	0.2370	0.2962	0.3555	0.4147	0.4739	0.5332	0.5924
2.0	0.0000	0.0592	0.1185	0.1777	0.2370	0.2962	0.3555(4)	0.4147	0.4739	0.5332	0.5924
						vertical displacement, y					
0.0	0.0897	0.1469	0.1764	0.1983	0.2153	0.2288	0.2393	0.2471	0.2525	0.2558	0.2568
0.2	0.3794	0.3812	0.3860	0.3923	0.3990	0.4053	0.4108	0.4152	0.4184	0.4204	0.4210
0.4	0.5836	0.5841	0.5853	0.5872	0.5894	0.5918	0.5940	0.5959	0.5974	0.5983	0.5986
0.6	0.7772	0.7774	0.7778	0.7784	0.7792	0.7801	0.7809	0.7816	0.7822	0.7826	0.7827
0.8	0.9676	0.9676	0.9678	0.9680	0.9683	0.9686	0.9689	0.9692	0.9694	0.9695	0.9696
1.0	1.1568	1.1568	1.1568	1.1569	1.1570	1.1571	1.1573	1.1574	1.1574	1.1575	1.1575
1.2	1.3456	1.3456	1.3456	1.3456	1.3457	1.3457	1.3458	1.3458	1.3458	1.3458	1.3459
1.4	1.5342	1.5342	1.5342	1.5343	1.5343	1.5343	1.5343	1.5343	1.5343	1.5343	1.5343
1.6	1.7228	1.7228	1.7228	1.7228	1.7229	1.7229	1.7229	1.7229	1.7229	1.7229	1.7229
1.8	1.9114	1.9114	1.9114	1.9114	1.9114	1.9114	1.9114	1.9114	1.9114	1.9114	1.9114
2.0	2.1000	2.1000	2.1000	2.1000	2.1000	2.1000	2.1000	2.1000	2.1000	2.1000	2.1000
						horizontal velocity, u					
0.0	0.0000	0.6843	0.8651	0.9877	1.0794	1.1497	1.2036	1.2435	1.2712	1.2874	1.2928
0.2	0.9259	0.9369	0.9654	1.0024	1.0407	1.0760	1.1061	1.1300	1.1473	1.1577	1.1612
0.4	1.0163	1.0189	1.0261	1.0368	1.0495	1.0628	1.0752	1.0858	1.0938	1.0988	1.1005
0.6	1.0449	1.0457	1.0480	1.0516	1.0561	1.0609	1.0656	1.0698	1.0730	1.0750	1.0758
0.8	1.0549	1.0552	1.0560	1.0573	1.0589	1.0607	1.0624	1.0640	1.0652	1.0660	1.0662
1.0	1.0585	1.0586	1.0589	1.0594	1.0600	1.0606	1.0613	1.0618	1.0623	1.0626	1.0627
1.2	1.0598	1.0599	1.0600	1.0602	1.0604	1.0606	1.0609	1.0611	1.0612	1.0613	1.0614
1.4	1.0603	1.0603	1.0604	1.0604	1.0605	1.0606	1.0607	1.0608	1.0608	1.0609	1.0609
1.6	1.0605	1.0605	1.0605	1.0605	1.0606	1.0606	1.0606	1.0607	1.0607	1.0607	1.0607
1.8	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0607	1.0607
2.0	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606	1.0606
						vertical velocity, v					
0.0	0.0000	0.3222	0.3444	0.3319	0.3024	0.2628	0.2165	0.1658	0.1120	0.0565	0.0000
0.2	0.0000	0.0486	0.0860	0.1079	0.1152	0.1108	0.0977	0.0782	0.0544	0.0279	0.0000
0.4	0.0000	0.0144	0.0269	0.0360	0.0411	0.0418	0.0385	0.0319	0.0227	0.0118	0.0000
0.6	0.0000	0.0049	0.0094	0.0128	0.0148	0.0154	0.0145	0.0122	0.0088	0.0046	0.0000
0.8	0.0000	0.0018	0.0034	0.0046	0.0054	0.0057	0.0054	0.0046	0.0033	0.0017	0.0000
1.0	0.0000	0.0006	0.0012	0.0017	0.0020	0.0021	0.0020	0.0017	0.0012	0.0006	0.0000
1.2	0.0000	0.0002	0.0005	0.0006	0.0007	0.0008	0.0007	0.0006	0.0005	0.0002	0.0000
1.4	0.0000	0.0001	0.0002	0.0002	0.0003	0.0003	0.0003	0.0002	0.0002	0.0001	0.0000
1.6	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000
1.8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

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TABLE 12*b*. TIME, PRESSURE AND ACCELERATION FOR $d = 10.0$ (ALSO APPLICABLE, WITH DIGITS IN PARENTHESIS, TO THE DEEP-WATER WAVE)

λ...	0.00	−0.05	−0.10	−0.15	−0.20	−0.25	−0.30	−0.35	−0.40	−0.45	−0.50
y						time, *t*					
0.0	0.0000	0.3165	0.4040	0.4683	0.5219	0.5693	0.6128	0.6537	0.6929	0.7311	0.7688
0.2	0.0000	0.0726	0.1419	0.2062	0.2657	0.3211	0.3734	0.4233	0.4716	0.5188	0.5654
0.4	0.0000	0.0607	0.1208	0.1799	0.2375	0.2938	0.3487	0.4024	0.4553	0.5075	0.5594
0.6	0.0000	0.0575	0.1149	0.1719	0.2284	0.2845	0.3401	0.3952	0.4499	0.5044	0.5587
0.8	0.0000	0.0565	0.1128	0.1691	0.2252	0.2812	0.3369	0.3925	0.4480	0.5033	0.5586
1.0	0.0000	0.0561	0.1121	0.1681	0.2241	0.2800	0.3358	0.3916	0.4473	0.5029	0.5586
1.2	0.0000	0.0559	0.1119	0.1678	0.2237	0.2795	0.3354	0.3912	0.4470	0.5028	0.5586
1.4	0.0000	0.0559	0.1118	0.1676	0.2235	0.2794	0.3352	0.3911	0.4469	0.5027	0.5586
1.6	0.0000	0.0559	0.1117	0.1676	0.2235	0.2793	0.3352	0.3910	0.4469	0.5027	0.5586
1.8	0.0000	0.0559	0.1117	0.1676	0.2234	0.2793	0.3351	0.3910	0.4469	0.5027	0.5586
2.0	0.0000	0.0559	0.1117	0.1676	0.2234	0.2793	0.3351	0.3910	0.4469	0.5027	0.5586
						pressure, *p*					
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	1.0200	1.0176	1.0116	1.0046	0.9982	0.9929	0.9889	0.9859	0.9839	0.9827	0.9823
0.4	1.9531	1.9526	1.9513	1.9493	1.9470	1.9447	1.9427	1.9410	1.9398	1.9390	1.9388
0.6	2.8917	2.8915	2.8911	2.8905	2.8897	2.8889	2.8881	2.8874	2.8868	2.8865	2.8864
0.8	3.8329	3.8328	3.8327	3.8325	3.8322	3.8319	3.8316	3.8313	3.8311	3.8310	3.8309
1.0	4.7751	4.7751	4.7751	4.7750	4.7749	4.7748	4.7747	4.7746	4.7745	4.7744	4.7744
1.2	5.7178	5.7178	5.7177	5.7177	5.7177	5.7176	5.7176	5.7176	5.7175	5.7175	5.7175
1.4	6.6605	6.6605	6.6605	6.6605	6.6605	6.6605	6.6605	6.6605	6.6604	6.6604	6.6604
1.6	7.6034	7.6034	7.6034	7.6033	7.6033	7.6033	7.6033	7.6033	7.6033	7.6033	7.6033
1.8	8.5462	8.5462	8.5462	8.5462	8.5462	8.5462	8.5462	8.5462	8.5462	8.5462	8.5462
2.0	9.4891(0)	9.4891(0)	9.4891(0)	9.4891(0)	9.4890	9.4890	9.4890	9.4890	9.4890	9.4890	9.4890
						horizontal acceleration					
0.0	0.0000	2.1216	1.9904	1.8110	1.5987	1.3625	1.1086	0.8418	0.5660	0.2844	0.0000
0.2	0.0000	0.2963	0.5117	0.6240	0.6499	0.6131	0.5326	0.4220	0.2915	0.1487	0.0000
0.4	0.0000	0.0831	0.1544	0.2055	0.2322	0.2344	0.2146	0.1768	0.1254	0.0649	0.0000
0.6	0.0000	0.0281	0.0530	0.0722	0.0837	0.0867	0.0813	0.0683	0.0491	0.0257	0.0000
0.8	0.0000	0.0100	0.0190	0.0261	0.0305	0.0319	0.0302	0.0256	0.0185	0.0097	0.0000
1.0	0.0000	0.0037	0.0069	0.0095	0.0112	0.0117	0.0111	0.0095	0.0069	0.0036	0.0000
1.2	0.0000	0.0013	0.0025	0.0035	0.0041	0.0043	0.0041	0.0035	0.0025	0.0013	0.0000
1.4	0.0000	0.0005	0.0009	0.0013	0.0015	0.0016	0.0015	0.0013	0.0009	0.0005	0.0000
1.6	0.0000	0.0002	0.0004	0.0005	0.0006	0.0006	0.0006	0.0005	0.0003	0.0002	0.0000
1.8	0.0000	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0000
2.0	0.0000	0.0000	0.0001(0)	0.0001	0.0002(1)	0.0002(1)	0.0002(1)	0.0001	0.0001(0)	0.0000	0.0000
						vertical acceleration					
0.0	2.4860	0.4938	0.0002	−0.3900	−0.7058	−0.9604	−1.1616	−1.3141	−1.4212	−1.4846	−1.5057
0.2	0.6926	0.6218	0.4454	0.2289	0.0167	−0.1709	−0.3265	−0.4477	−0.5340	−0.5857	−0.6029
0.4	0.2421	0.2270	0.1847	0.1225	0.0499	−0.0243	−0.0930	−0.1508	−0.1943	−0.2212	−0.2303
0.6	0.0876	0.0829	0.0695	0.0490	0.0238	−0.0033	−0.0297	−0.0528	−0.0708	−0.0821	−0.0860
0.8	0.0320	0.0304	0.0257	0.0185	0.0095	−0.0004	−0.0103	−0.0190	−0.0259	−0.0303	−0.0318
1.0	0.0118	0.0112	0.0095	0.0069	0.0036	−0.0001	−0.0037	−0.0069	−0.0095	−0.0112	−0.0117
1.2	0.0043	0.0041	0.0035	0.0025	0.0013	0.0000	−0.0013	−0.0025	−0.0035	−0.0041	−0.0043
1.4	0.0016	0.0015	0.0013	0.0009	0.0005	0.0000	−0.0005	−0.0009	−0.0013	−0.0015	−0.0016
1.6	0.0006	0.0005	0.0005	0.0003	0.0002	0.0000	−0.0002	−0.0003	−0.0005	−0.0005	−0.0006
1.8	0.0002	0.0002	0.0002	0.0001	0.0001	0.0000	−0.0001	−0.0001	−0.0002	−0.0002	−0.0002
2.0	0.0000(1)	0.0000(1)	0.0000(1)	0.0000	0.0000	0.0000	0.0000	0.0000	−0.0000(1)	−0.0000(1)	−0.0000(1)

Table 12c. Displacement and velocity near the surface for d = 10.0 (also applicable to the deep-water wave)

/λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ	horizontal displacement, x											
0.00	0.0000	0.0017	0.0026	0.0049	0.0077	0.0101	0.0143	0.0227	0.0362	0.0529	0.0674	0.5924
0.01	0.0000	0.0003	0.0006	0.0015	0.0030	0.0045	0.0075	0.0148	0.0281	0.0454	0.0606	0.5924
0.02	0.0000	0.0002	0.0005	0.0012	0.0024	0.0037	0.0061	0.0122	0.0239	0.0403	0.0553	0.5924
0.03	0.0000	0.0002	0.0004	0.0011	0.0022	0.0033	0.0054	0.0108	0.0214	0.0368	0.0513	0.5924
0.04	0.0000	0.0002	0.0004	0.0010	0.0020	0.0030	0.0050	0.0100	0.0198	0.0343	0.0482	0.5924
0.05	0.0000	0.0002	0.0004	0.0009	0.0019	0.0028	0.0047	0.0094	0.0187	0.0324	0.0458	0.5924
0.06	0.0000	0.0002	0.0004	0.0009	0.0018	0.0027	0.0045	0.0089	0.0178	0.0309	0.0439	0.5924
0.07	0.0000	0.0002	0.0003	0.0009	0.0017	0.0026	0.0043	0.0086	0.0171	0.0298	0.0423	0.5924
0.08	0.0000	0.0002	0.0003	0.0008	0.0017	0.0025	0.0041	0.0083	0.0165	0.0288	0.0410	0.5924
0.09	0.0000	0.0002	0.0003	0.0008	0.0016	0.0024	0.0040	0.0080	0.0161	0.0280	0.0399	0.5924
0.10	0.0000	0.0002	0.0003	0.0008	0.0016	0.0024	0.0039	0.0078	0.0157	0.0273	0.0390	0.5924
	vertical displacement, y											
0.00	0.0897	0.0907	0.0912	0.0925	0.0942	0.0955	0.0979	0.1026	0.1101	0.1191	0.1267	0.2568
0.01	0.1256	0.1256	0.1256	0.1256	0.1257	0.1257	0.1260	0.1270	0.1301	0.1359	0.1416	0.2646
0.02	0.1471	0.1471	0.1471	0.1471	0.1471	0.1471	0.1472	0.1476	0.1492	0.1527	0.1569	0.2724
0.03	0.1653	0.1653	0.1653	0.1653	0.1654	0.1654	0.1654	0.1657	0.1666	0.1690	0.1720	0.2803
0.04	0.1819	0.1819	0.1819	0.1819	0.1819	0.1819	0.1819	0.1821	0.1827	0.1844	0.1867	0.2882
0.05	0.1972	0.1972	0.1972	0.1972	0.1972	0.1972	0.1973	0.1974	0.1979	0.1991	0.2009	0.2962
0.06	0.2118	0.2118	0.2118	0.2118	0.2118	0.2118	0.2118	0.2119	0.2123	0.2132	0.2147	0.3042
0.07	0.2257	0.2257	0.2257	0.2257	0.2257	0.2257	0.2257	0.2258	0.2261	0.2269	0.2280	0.3123
0.08	0.2391	0.2391	0.2391	0.2391	0.2391	0.2391	0.2391	0.2391	0.2394	0.2400	0.2410	0.3204
0.09	0.2520	0.2520	0.2520	0.2520	0.2520	0.2520	0.2520	0.2521	0.2523	0.2529	0.2537	0.3285
0.10	0.2647	0.2647	0.2647	0.2647	0.2647	0.2647	0.2647	0.2647	0.2649	0.2654	0.2661	0.3367
	horizontal velocity, u											
0.00	0.0000	0.0849	0.1070	0.1453	0.1831	0.2097	0.2488	0.3139	0.3963	0.4788	0.5404	1.2928
0.01	0.4133	0.4133	0.4133	0.4135	0.4141	0.4150	0.4178	0.4298	0.4642	0.5186	0.5674	1.2839
0.02	0.5130	0.5130	0.5130	0.5131	0.5133	0.5135	0.5145	0.5187	0.5339	0.5659	0.6010	1.2753
0.03	0.5795	0.5795	0.5795	0.5795	0.5796	0.5797	0.5802	0.5824	0.5909	0.6109	0.6358	1.2670
0.04	0.6299	0.6299	0.6299	0.6299	0.6300	0.6301	0.6304	0.6318	0.6372	0.6507	0.6688	1.2589
0.05	0.6706	0.6706	0.6706	0.6707	0.6707	0.6708	0.6710	0.6719	0.6757	0.6855	0.6991	1.2511
0.06	0.7047	0.7047	0.7047	0.7047	0.7048	0.7048	0.7050	0.7057	0.7085	0.7158	0.7264	1.2435
0.07	0.7339	0.7339	0.7339	0.7339	0.7340	0.7340	0.7341	0.7347	0.7368	0.7426	0.7511	1.2362
0.08	0.7594	0.7594	0.7594	0.7594	0.7594	0.7594	0.7595	0.7600	0.7617	0.7664	0.7732	1.2291
0.09	0.7819	0.7819	0.7819	0.7819	0.7819	0.7819	0.7820	0.7823	0.7838	0.7876	0.7933	1.2222
0.10	0.8019	0.8019	0.8019	0.8019	0.8019	0.8019	0.8020	0.8023	0.8035	0.8067	0.8115	1.2156
	vertical velocity, v											
0.00	0.0000	0.0490	0.0617	0.0835	0.1049	0.1197	0.1411	0.1756	0.2167	0.2538	0.2783	0.0000
0.01	0.0000	0.0016	0.0033	0.0082	0.0164	0.0245	0.0404	0.0769	0.1335	0.1887	0.2244	0.0000
0.02	0.0000	0.0010	0.0020	0.0049	0.0098	0.0147	0.0245	0.0482	0.0915	0.1433	0.1815	0.0000
0.03	0.0000	0.0007	0.0014	0.0036	0.0072	0.0107	0.0178	0.0354	0.0690	0.1132	0.1492	0.0000
0.04	0.0000	0.0006	0.0011	0.0028	0.0056	0.0085	0.0141	0.0281	0.0552	0.0926	0.1251	0.0000
0.05	0.0000	0.0005	0.0009	0.0023	0.0047	0.0070	0.0116	0.0232	0.0458	0.0779	0.1068	0.0000
0.06	0.0000	0.0004	0.0008	0.0020	0.0039	0.0059	0.0099	0.0197	0.0390	0.0668	0.0925	0.0000
0.07	0.0000	0.0003	0.0007	0.0017	0.0034	0.0051	0.0085	0.0170	0.0339	0.0582	0.0811	0.0000
0.08	0.0000	0.0003	0.0006	0.0015	0.0030	0.0045	0.0075	0.0150	0.0298	0.0514	0.0719	0.0000
0.09	0.0000	0.0003	0.0005	0.0013	0.0027	0.0040	0.0066	0.0133	0.0265	0.0458	0.0643	0.0000
0.10	0.0000	0.0002	0.0005	0.0012	0.0024	0.0036	0.0060	0.0119	0.0237	0.0411	0.0578	0.0000

Table 12*d*. Time, pressure and acceleration near the surface for $d = 10.0$ (also applicable to the deep-water wave)

/λ...	0.0000	−0.0001	−0.0002	−0.0005	−0.0010	−0.0015	−0.0025	−0.0050	−0.0100	−0.0175	−0.0250	−0.5000
ψ						time, t						
0.00	0.0000	0.0393	0.0495	0.0672	0.0846	0.0969	0.1149	0.1449	0.1829	0.2210	0.2494	0.7688
0.01	0.0000	0.0007	0.0015	0.0037	0.0073	0.0110	0.0182	0.0354	0.0653	0.1005	0.1284	0.6549
0.02	0.0000	0.0005	0.0010	0.0024	0.0048	0.0072	0.0119	0.0236	0.0460	0.0758	0.1015	0.6322
0.03	0.0000	0.0004	0.0007	0.0019	0.0037	0.0056	0.0093	0.0186	0.0368	0.0623	0.0856	0.6183
0.04	0.0000	0.0003	0.0006	0.0016	0.0032	0.0047	0.0079	0.0158	0.0313	0.0538	0.0749	0.6084
0.05	0.0000	0.0003	0.0006	0.0014	0.0028	0.0042	0.0070	0.0139	0.0278	0.0480	0.0673	0.6009
0.06	0.0000	0.0003	0.0005	0.0013	0.0025	0.0038	0.0063	0.0126	0.0252	0.0437	0.0616	0.5950
0.07	0.0000	0.0002	0.0005	0.0012	0.0023	0.0035	0.0058	0.0117	0.0233	0.0404	0.0572	0.5902
0.08	0.0000	0.0002	0.0004	0.0011	0.0022	0.0033	0.0054	0.0109	0.0217	0.0378	0.0537	0.5862
0.09	0.0000	0.0002	0.0004	0.0010	0.0021	0.0031	0.0051	0.0103	0.0205	0.0358	0.0508	0.5828
0.10	0.0000	0.0002	0.0004	0.0010	0.0020	0.0029	0.0049	0.0098	0.0195	0.0340	0.0484	0.5800
						pressure, p						
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.01	0.0941	0.0941	0.0941	0.0940	0.0939	0.0938	0.0932	0.0911	0.0856	0.0785	0.0735	0.0502
0.02	0.1553	0.1553	0.1553	0.1553	0.1552	0.1552	0.1549	0.1540	0.1509	0.1448	0.1391	0.1003
0.03	0.2103	0.2103	0.2103	0.2103	0.2103	0.2103	0.2102	0.2096	0.2077	0.2033	0.1984	0.1503
0.04	0.2624	0.2624	0.2624	0.2624	0.2624	0.2624	0.2623	0.2620	0.2607	0.2575	0.2535	0.2001
0.05	0.3128	0.3128	0.3128	0.3128	0.3128	0.3127	0.3127	0.3125	0.3115	0.3092	0.3060	0.2497
0.06	0.3620	0.3620	0.3620	0.3620	0.3620	0.3620	0.3620	0.3618	0.3611	0.3592	0.3567	0.2993
0.07	0.4105	0.4105	0.4105	0.4105	0.4105	0.4105	0.4104	0.4103	0.4098	0.4083	0.4063	0.3487
0.08	0.4584	0.4584	0.4584	0.4584	0.4584	0.4584	0.4584	0.4583	0.4578	0.4567	0.4550	0.3980
0.09	0.5060	0.5060	0.5060	0.5060	0.5060	0.5060	0.5060	0.5059	0.5055	0.5045	0.5031	0.4472
0.10	0.5532	0.5532	0.5532	0.5532	0.5532	0.5532	0.5532	0.5531	0.5528	0.5520	0.5509	0.4962
						horizontal acceleration						
0.00	0.0000	2.1641	2.1662	2.1686	2.1698	2.1701	2.1699	2.1691	2.1686	2.1667	2.1610	0.0000
0.01	0.0000	0.0204	0.0407	0.1016	0.2022	0.3010	0.4897	0.8878	1.3712	1.6797	1.8103	0.0000
0.02	0.0000	0.0099	0.0199	0.0496	0.0992	0.1484	0.2458	0.4779	0.8662	1.2491	1.4676	0.0000
0.03	0.0000	0.0065	0.0129	0.0323	0.0646	0.0968	0.1610	0.3177	0.6051	0.9465	1.1846	0.0000
0.04	0.0000	0.0047	0.0095	0.0237	0.0474	0.0710	0.1181	0.2345	0.4556	0.7425	0.9674	0.0000
0.05	0.0000	0.0037	0.0074	0.0185	0.0370	0.0555	0.0924	0.1838	0.3607	0.6012	0.8034	0.0000
0.06	0.0000	0.0030	0.0060	0.0151	0.0301	0.0451	0.0752	0.1498	0.2956	0.4993	0.6785	0.0000
0.07	0.0000	0.0025	0.0050	0.0126	0.0252	0.0378	0.0629	0.1255	0.2484	0.4232	0.5814	0.0000
0.08	0.0000	0.0022	0.0043	0.0108	0.0215	0.0322	0.0537	0.1072	0.2127	0.3644	0.5046	0.0000
0.09	0.0000	0.0019	0.0037	0.0093	0.0186	0.0280	0.0466	0.0930	0.1848	0.3179	0.4427	0.0000
0.10	0.0000	0.0016	0.0033	0.0082	0.0164	0.0245	0.0409	0.0817	0.1624	0.2802	0.3918	0.0000
						vertical acceleration						
0.00	2.4860	1.2451	1.2415	1.2308	1.2152	1.2013	1.1765	1.1232	1.0325	0.9129	0.8044	−1.5057
0.01	2.2392	2.2391	2.2388	2.2368	2.2298	2.2182	2.1831	2.0481	1.7407	1.3979	1.1658	−1.4407
0.02	2.0615	2.0614	2.0614	2.0608	2.0590	2.0558	2.0460	2.0024	1.8588	1.6036	1.3743	−1.3782
0.03	1.9124	1.9124	1.9124	1.9121	1.9113	1.9098	1.9052	1.8842	1.8076	1.6413	1.4590	−1.3182
0.04	1.7820	1.7820	1.7820	1.7818	1.7813	1.7805	1.7778	1.7655	1.7186	1.6068	1.4691	−1.2605
0.05	1.6655	1.6655	1.6655	1.6654	1.6651	1.6645	1.6628	1.6546	1.6230	1.5441	1.4398	−1.2052
0.06	1.5601	1.5601	1.5601	1.5601	1.5598	1.5594	1.5582	1.5524	1.5297	1.4713	1.3909	−1.1520
0.07	1.4640	1.4640	1.4640	1.4640	1.4638	1.4635	1.4626	1.4582	1.4411	1.3964	1.3330	−1.1011
0.08	1.3758	1.3758	1.3758	1.3757	1.3756	1.3754	1.3747	1.3713	1.3579	1.3226	1.2716	−1.0522
0.09	1.2944	1.2944	1.2944	1.2944	1.2943	1.2941	1.2935	1.2908	1.2801	1.2516	1.2097	−1.0053
0.10	1.2191	1.2191	1.2191	1.2190	1.2189	1.2188	1.2183	1.2161	1.2073	1.1838	1.1490	−0.9603

Table 12*e*. Mean depth as a function of ψ for $d = 10.0$

$\psi = 0(0.01)0.1$	1.8978	1.8859	1.8753	1.8648	1.8545	1.8443	1.8343	1.8243	1.8143	1.8045	1.7947
$\psi = 0(0.2)2.0$	1.8978	1.6983	1.5087	1.3200	1.1314	0.9429	0.7543	0.5657	0.3771	0.1886	0.0000

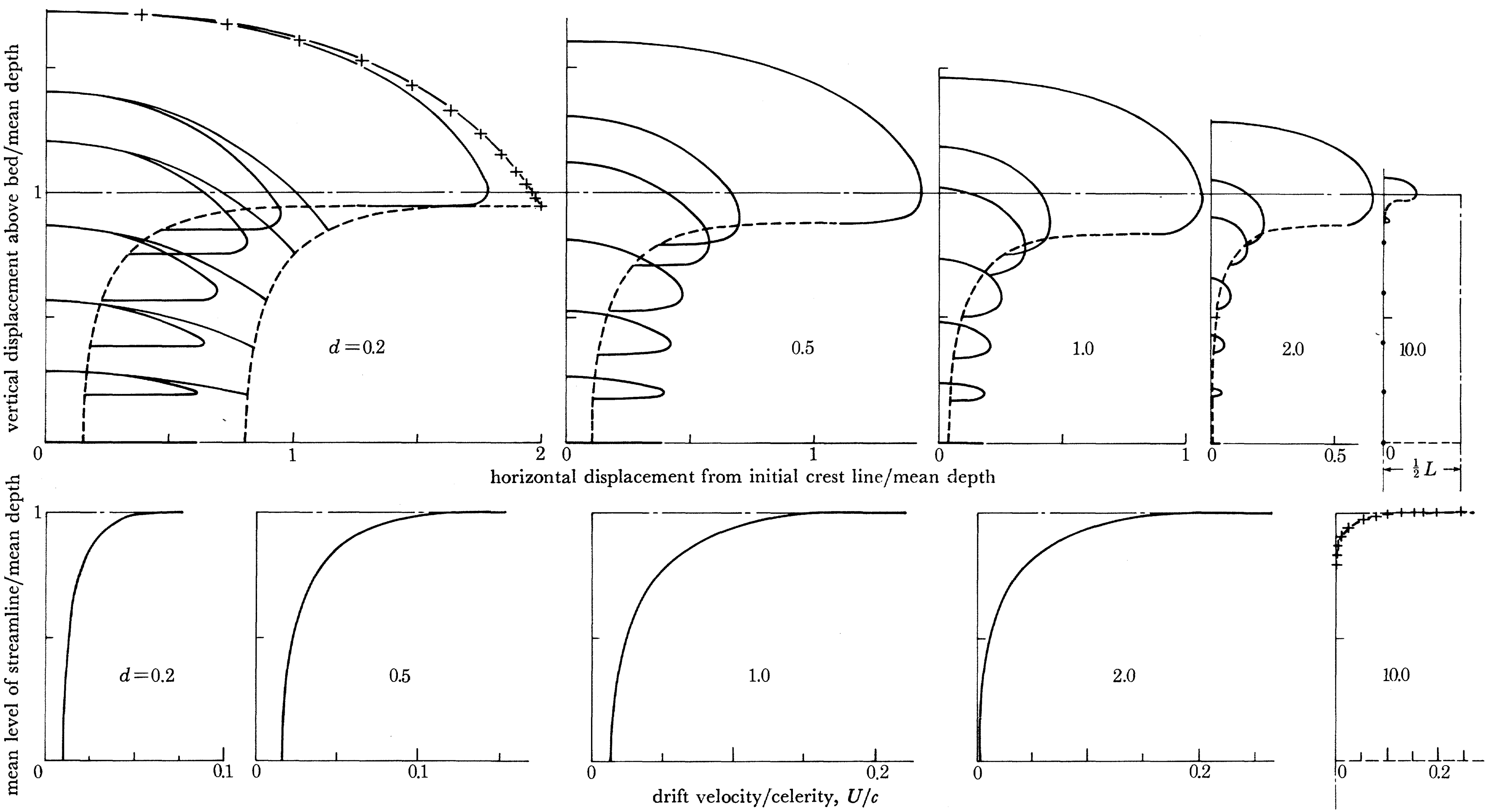

FIGURE 5. Particle semi-orbits (upper diagrams) and drift profiles (lower diagrams) for the cases presented in tables 8–12, plotted to a common mean depth and gravity. Orbits are plotted for $\psi = 0$ (surface), -0.2, -0.4, -0.8, -1.2, -1.6, -2.0 (bed). Orbits for the solitary wave are shown with those for $d = 0.2$; orbits and drift profile for $d = 10.0$ apply also to the deep-water wave. +, Predictions of Longuet-Higgins (1979) from a simplified analysis for the solitary wave and deep-water wave.

profiles on to the perimeter of a hexagon. His predicted drift profile is plotted in figure 5 and again shows very close agreement. The drift velocity at the surface is $0.2734c$, compared with $0.274c$ as predicted by Longuet-Higgins from the results of Yamada & Schwartz.

The drift profiles confirm the very strong drift gradient near the surface which was pointed out and explained by Longuet-Higgins. In any steady inviscid flow that includes a stagnation point, particles on the stagnation-point streamline will have considerably longer travelling times and will lag behind particles on adjacent streamlines, as is made clear by the tabulated times in tables 8–12. When a celerity opposed to the direction of steady motion is superimposed these particles will lead instead of lagging and will give the strong forward drift which has been found. When stagnation or near-stagnation conditions are not present a much more uniform behaviour is to be expected; consequently the strong surface drift gradient is a feature only of waves at or very near to the maximum height.

13. Discussion

The solutions obtained have been demonstrated to have inherent high accuracy and to be consistent with previous accurate results at the two extremes of the depth: wavelength-ratio range. They may thus be fairly put forward as definitive solutions, not previously available, whose accuracy exceeds that required in almost all practical applications. They do not, however, constitute a mere academic curiosity to the practising coastal engineer or oceanographer who will now be able to work to, say, three or four decimals in full knowledge of the error thus incurred. Furthermore for some applications, such as the calculation of the level of action of the maximum wave, the accuracy achieved is still only marginally sufficient, as has been demonstrated.

Theoretically, the results should provide a useful background to studies of the exact nature of the singularities at the wave crest and other cases of flow at a corner.

Of equal importance to the generation of the results is their presentation in a readily usable form. It is hoped that the limited number of sets of coefficients and detailed tabulations included herein will encourage and facilitate further theoretical study and early practical application.

The method can in principle be extended to waves of less than maximum height and further work is in progress to achieve this. Grant (1973) and Schwartz (1974) have shown that for such waves the singularity not only moves away from the crest in the τ-plane but also changes in order from $\frac{2}{3}$ towards $\frac{1}{2}$. Moderately accurate solutions have been readily obtained in this way and the aim is now to refine the strategy to preserve the high accuracy obtained for the maximum waves. A valuable analysis of the problem has been provided by Longuet-Higgins & Fox (1977, 1978) while solutions for comparison are available from the work of Sasaki & Murakami (1973) and Byatt-Smith & Longuet-Higgins (1976). The extension of the work to near-maximum waves is particularly desirable in view of recent demonstrations, for example by Cokelet (1977), that the speed, energy and other properties reach maxima for waves a little below the maximum amplitude.

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Appendix 1. Further terms in Grant's expansion for the flow near an angled crest

Grant's expansion (3.4) may be extended in the form

$$Z = -\mathrm{i}(\surd 2F)^{\frac{2}{3}}\left[-(\tfrac{3}{2})^{\frac{2}{3}}(\mathrm{i}\chi)^{\frac{2}{3}} + \sum_{r=1}^{\infty} b_r(\mathrm{i}\chi)^{[r(\mu-\frac{2}{3})+\frac{2}{3}]}\right], \qquad (\mathrm{A}\,1.1)$$

where $\mu = 1.469\,345\,741$, the first root of (3.5) greater than $\frac{2}{3}$.

If we construct the product $\mathrm{Im}\,(Z)|\mathrm{d}Z/\mathrm{d}\chi|^2$, $\psi = 0$, and equate to zero the coefficients of $\phi^{r(\mu-\frac{2}{3})}$, $r = 2, 3, \ldots, 9$, we may express each b_r in terms of b_1, according to table A 1.

Table A1. Exponents and coefficients in Grant's expansion, in the form (A 1.1)

r	$r(\mu-\frac{2}{3})+\frac{2}{3}$	b_r/b_1^r
1	1.469 345 741	1.0
2	2.272 024 815	−0.315 904 354
3	3.074 703 889	−0.208 228 586
4	3.877 382 963	−0.018 574 115
5	4.680 062 037	0.376 381 114
6	5.482 741 111	−0.732 454 738
7	6.285 420 185	0.410 851 357
8	7.088 099 259	0.310 884 668
9	7.890 778 333	−0.015 452 448

Norman (1974) has also developed an expansion that is in effect of the form (A 1.1), expressing his resulting coefficients as rational functions of a quantity related to μ. His published results allow a direct check to be made only as far as b_3, up to which point both expansions agree.

Appendix 2. Expansion for evaluating t on the surface streamline near the crest

We require
$$t(1,\theta) = \tfrac{1}{2}d\int_0^\theta |\tau\,\mathrm{d}z/\mathrm{d}\tau|^2_{\rho=1}\,\mathrm{d}\theta,$$

where θ is small. With the aid of (2.6), (2.10), (C) and (3.7) z is expanded as a series in θ. The integral is set up and integrated term by term to include terms up to order $\theta^{\mu+1}$.

We define
$$\begin{aligned} L' &= L/2\pi = (2/d)\,(1+\tfrac{1}{2}a_0),\\ \tilde{R} &= R^2/(1-R^2),\\ s' &= \tfrac{2}{3}s/(1-R^2)^{\frac{2}{3}},\\ q' &= \mu q/(1-R^2)^{\mu}, \quad \text{where} \quad \mu = 1.469\,345\,741,\\ k_1 &= -\tfrac{2}{3}s\tilde{R}[1+\tfrac{1}{3}\tilde{R}] - \mu q\tilde{R}[1+(1-\mu)\,\tilde{R}] + \sum_{m=1}^{N-2} m^2 a_m,\\ k_2 &= -L' - \tfrac{2}{3}s\tilde{R} - \mu q\tilde{R} - \sum_{m=1}^{N-2} m a_m \coth md. \end{aligned}$$

The required expansion may then be written

$$\begin{aligned} 2t/d = {} & 3s'^2\theta^{\frac{1}{3}} - \tfrac{3\sqrt{3}}{2}s'k_2\theta^{\frac{2}{3}} + k_2^2\,\theta \\ & -[2/(\mu-\tfrac{1}{3})]\,s'q'\sin[\tfrac{1}{2}\pi(\mu-\tfrac{5}{3})]\,\theta^{\mu-\frac{1}{3}} \\ & -(2/\mu)\,q'k_2\sin\tfrac{1}{2}\pi\mu\;\theta^{\mu} + \tfrac{3}{5}s'(k_1+\tfrac{5}{6}k_2)\,\theta^{\frac{5}{3}} \\ & +[1/(2\mu-1)]\,q'^2\theta^{2\mu-1} \\ & +[(\mu-\tfrac{2}{3})/(\mu+\tfrac{2}{3})]\,s'q'\sin[\tfrac{1}{2}\pi(\mu-\tfrac{2}{3})]\,\theta^{\mu+\frac{2}{3}} \\ & +\tfrac{1}{84}s'^2\theta^{\frac{7}{3}} \\ & +[2/(\mu+1)]\,q'[k_1+\tfrac{1}{2}(\mu+1)\,k_2]\cos\tfrac{1}{2}\pi\mu\;\theta^{\mu+1}. \end{aligned}$$

This expansion has been used to compute t for $0 < \theta/2\pi \leqslant 0.0025$ on the surface streamline.

For the Stokes corner flow s' has a theoretical value of $(4F/\sqrt{3}\,d)^{\frac{2}{3}}$ which follows from (3.2) and (3.7). However, as explained in §6, s was allowed to float in the iteration in the interests of obtaining a solution of better overall accuracy. The computed crest acceleration therefore departs from its theoretical value (see Longuet-Higgins & Fox 1977) of $\frac{1}{2}g$, or $1/4F^2$ in our notation, and imposes a consequential error on t near the crest. In all solutions the acceleration shows a negative error, giving a positive error in t.

To consider conditions on the surface streamline, the acceleration is significantly in error only over a limited zone between $\theta = 0$ and $\theta/2\pi < 0.0001$ in all solutions. In this zone there is a corresponding large positive gradient of p_s which, however, must reverse to restore p_s to zero at the second nodal point, $\theta = \theta_c = \frac{2}{560}\pi$ in most solutions. We therefore consider the error which occurs in t at $\theta/2\pi = 0.0001$, the first tabulated point in tables 8 to 12. At this point t is almost entirely accounted for by the first term of the above expansion.

At the shallow-water end of the range, $d = 0.2$, the computed s differs from the theoretical value by 0.033 %. The expected error in t is therefore 0.066 % of the computed value of 0.6653 (table 8), or 0.0004.

At the deep-water end, $d = 10$, the error in s is 0.19 % and, from table 12, the resultant error in t is 0.38 % of 0.0393, or 0.0002.

These estimated errors are considered to be upper bounds because the artificial negative gradient of p_s following this zone will serve to introduce some self-correction in t at greater values of θ.

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